The Special Theory of Relativity explained to children

(from 7 to 107 years old)

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Prologue

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This is the discussion I would like to have with her (or him).

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— Time and space seem to me very intuitive, and yet difficult to understand in deep . . .

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We should keep a modest mind profile on such a subject. We cannot hope to understand all the mysteries of Time and Space. We should only try to understand some of their properties and to use them to describe physical phenomena, and we should be ready to change the way we think about Time and Space, if some experimental evidence shows that we were wrong.

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Let me now indicate how the mental pictures of Time and Space used by scientists have evolved, mainly from Newton to Einstein.

The absolute Time of Newton

The great scientist Isaac Newton (1642–1727) used, as mental picture of Time, a straight line $\mathcal{T}$, going to infinity on both sides, hence without beginning nor end, with no privileged origin. Each particular time, for example “now”, or ”three days ago at the sunset at Paris”, corresponds to a particular element of that straight line.

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Observe that Newton considered, without any discussion, that for each event happening in the universe, there was a corresponding well defined time (element of the straight line $\mathcal{T}$), the time at which that event happens.

— Where is that straight line $\mathcal{T}$? Is it drawn in some plane or in space?

The absolute Time of Newton (2)

— Nowhere! You should not think about the straight line of Time $\mathcal{T}$ as drawn in something of larger dimension. Newton considered Time as an abstract straight line, because successive events are linearly ordered, like points on a straight line. Don't forget that $\mathcal{T}$ is a mental picture of Time, not Time itself! However, that mental picture is much more than a confuse idea: it has very well defined mathematical properties. In modern language, we say that $\mathcal{T}$ is endowed with an affine structure and with an orientation.

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— What is an affine structure? and what is its use?

— An affine structure allows us to compare two time intervals and to take their ratio, for example to say that one of these intervals is two times the other one.

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For the physicist, it means that the physical laws remain the same at all times.

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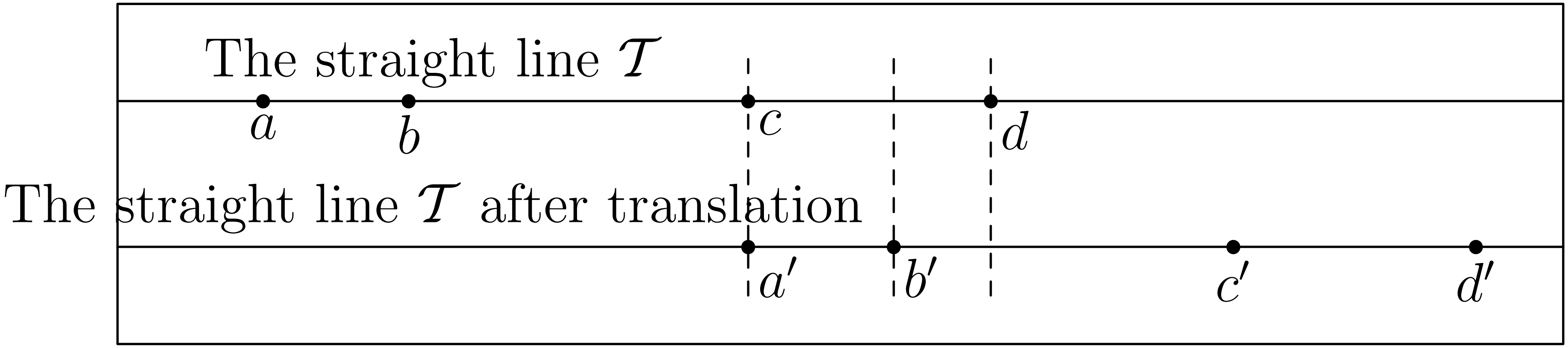

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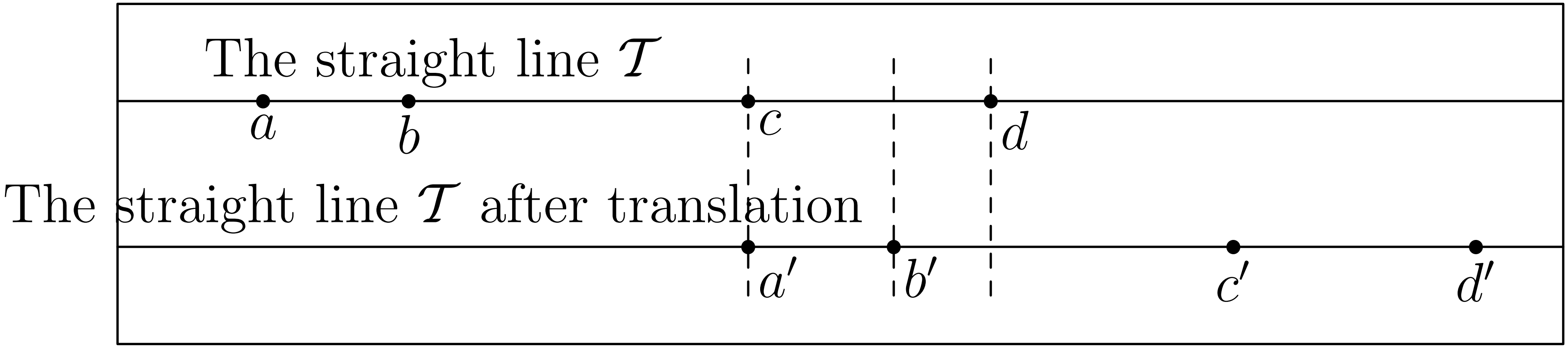

Other important property of Time: it always flows from past to future. To take it into account, we endow $\mathcal{T}$ with an orientation; it means that we consider the two directions (from past to future and from future to past) as different, not equivalent, for example by choosing the direction from past to future as preferred. We then say that $\mathcal{T}$ is oriented.

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For Newton, every object A of the physical world occupies, at each time t (element of $\mathcal{T}$) for which that object exists, a position A_t in Space $\mathcal{E}$. The motion of of A is described by its successive positions A_t when t varies in $\mathcal{T}$.

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Therefore I use it now, with the absolute Time and Space of Newton, although Newton itself did not use that concept.

Newton's Space-Time

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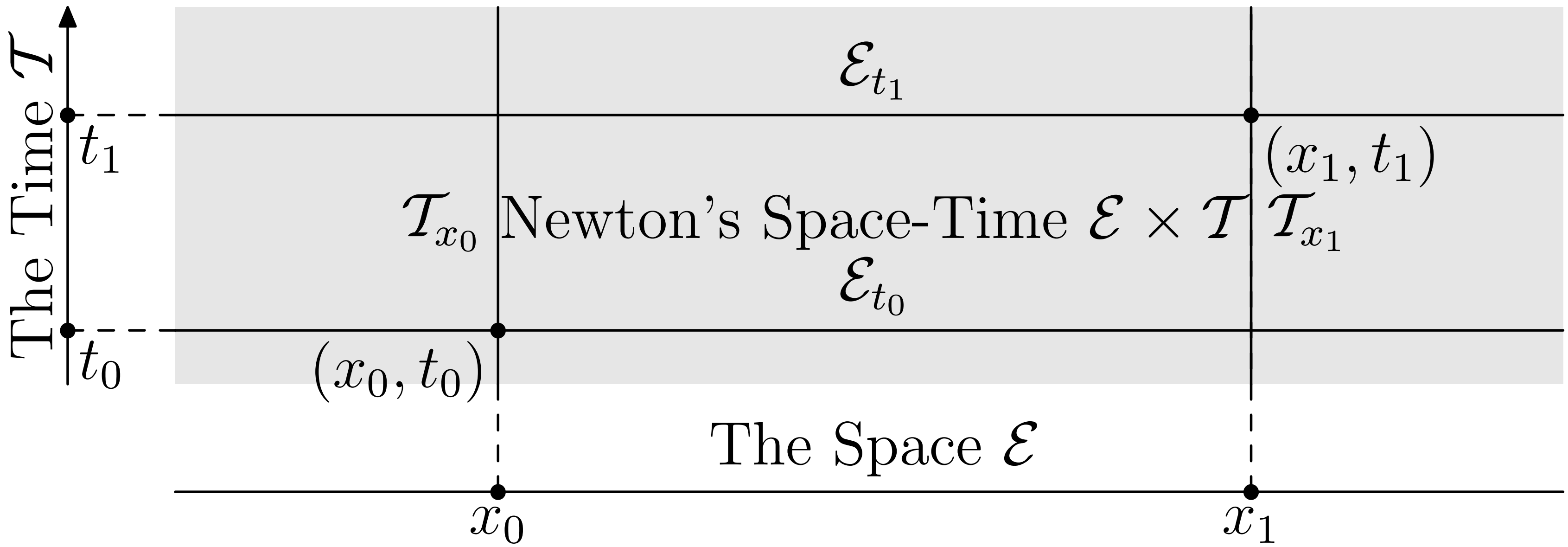

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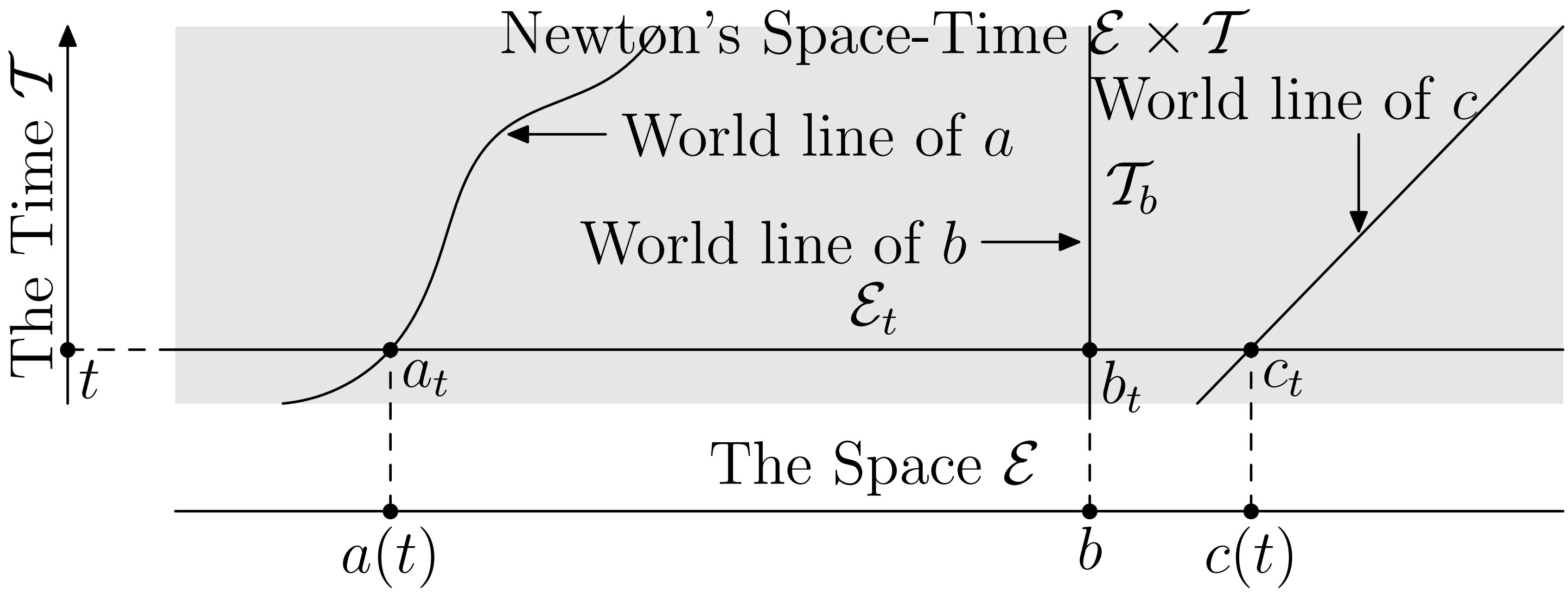
You will see on the next picture the world lines of three particles, a, b and c.

World lines



Newton's Space-Time $\mathcal{E} \times \mathcal{T}$
The Time $\mathcal{T}$
World line of a
World line of b
World line of c
$\mathcal{T}_b$
$\mathcal{E}_t$
t
a_t
b_t
c_t
The Space $\mathcal{E}$
$a(t)$
b
$c(t)$

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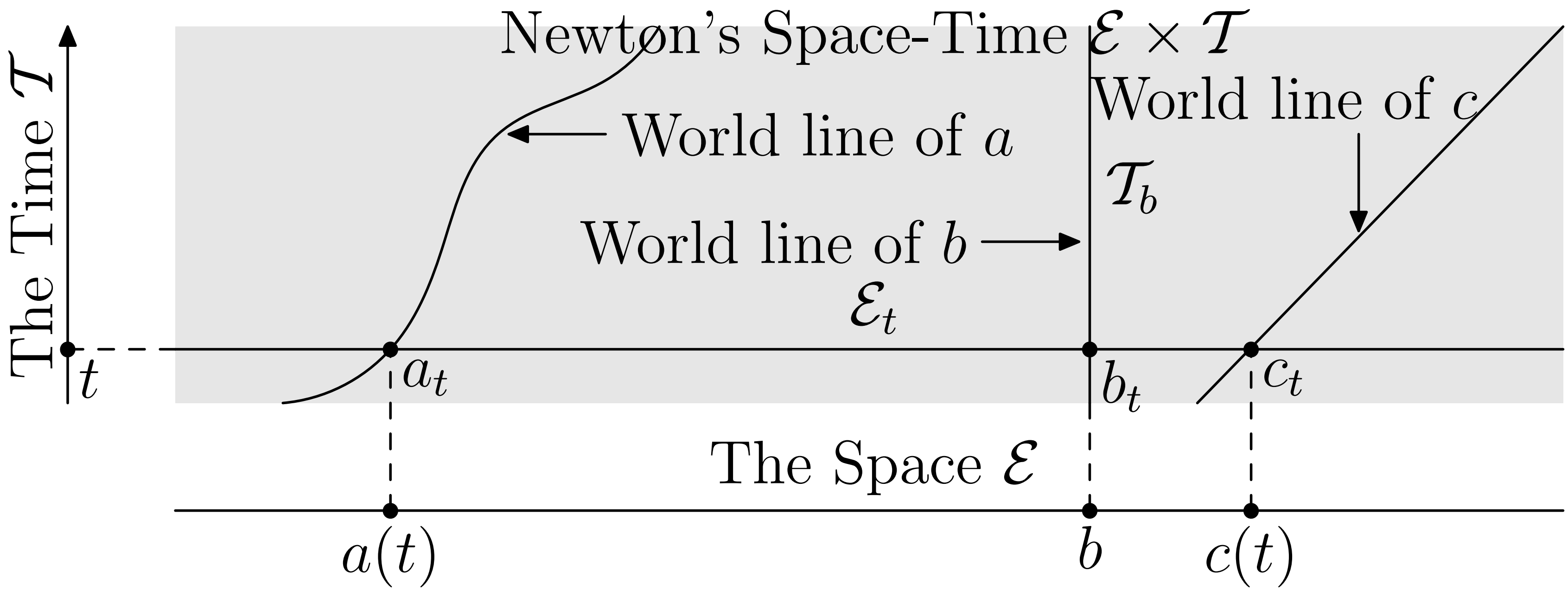

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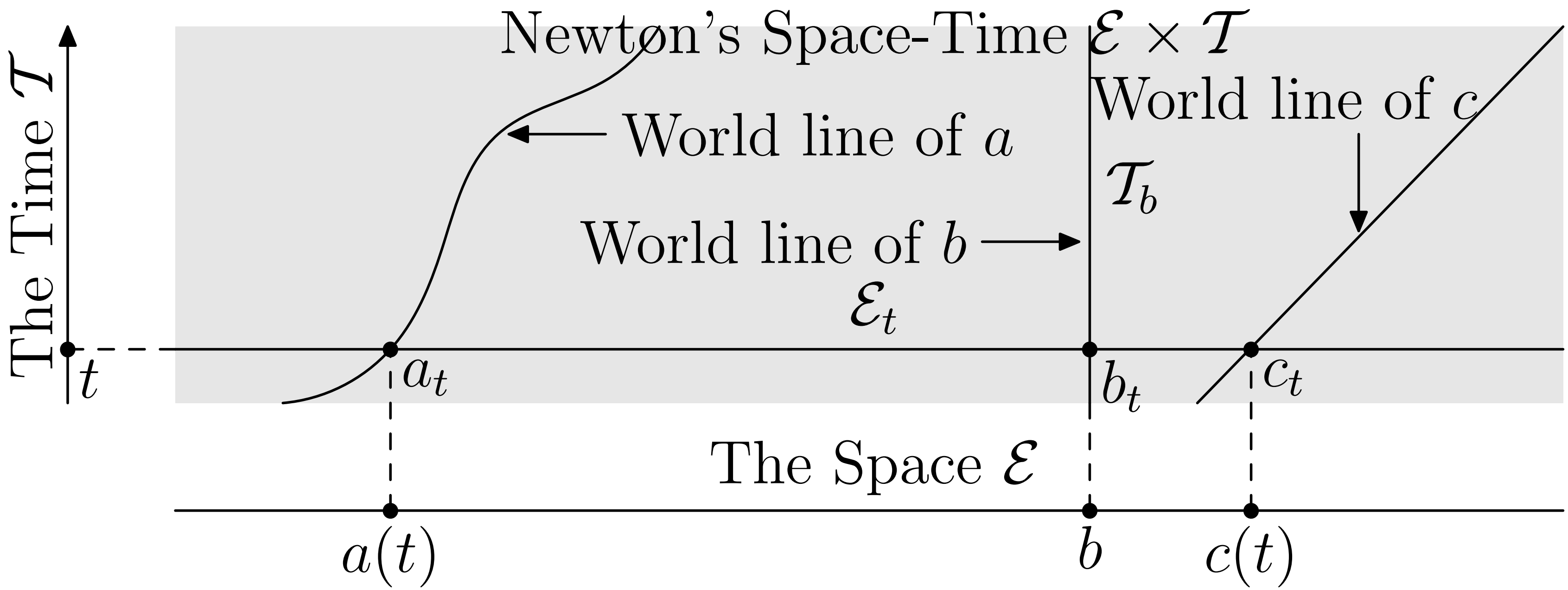

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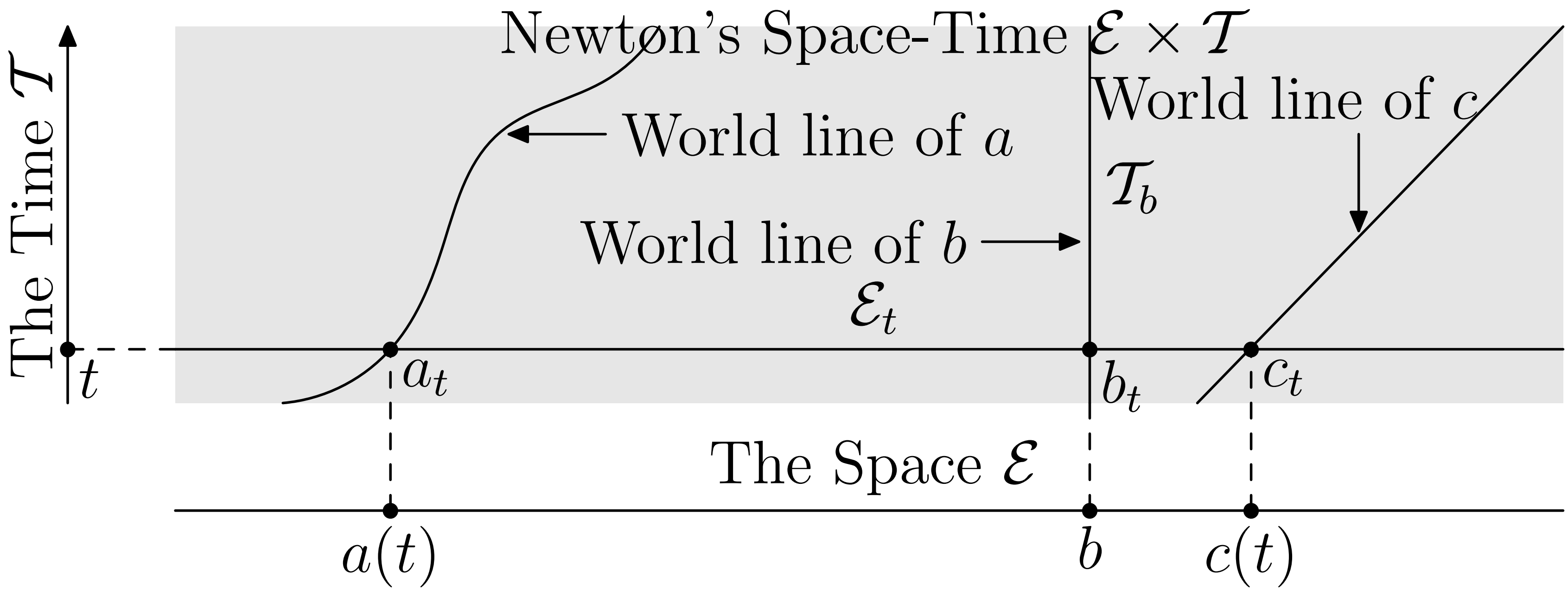

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The world line of a is a curve, not a straight line. It means that the velocity of a changes with time.

Absolute rest and motion

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— This seems very natural. Why should we change this view?

— Because nothing is at rest in the Universe! The Earth rotates around its axis and around the Sun, which rotates around the center of our Galaxy. And there are billions of galaxies in the Universe, all moving with respect to the others! For these reasons, Newton's concept of an absolute Space was criticized very early, notably by his contemporary, the great mathematician and philosopher Gottfried Wilhelm Leibniz (1647–1716).

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— To study the motion of a material body A (for example a falling apple, the Earth or the Mars planet), Newton, and after him almost all scientists up to now, used a reference frame. It means that he used a body R which remained approximately rigid during the motion he wanted to study, and he made as if that body was at rest. Then he could study the relative motion of A with respect to R.

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Assuming that Newton's absolute Space $\mathcal{E}$ exists, we recover the description of absolute motion of A by choosing, for R, a body at rest in $\mathcal{E}$. The corresponding reference frame is called the absolute fixed frame.

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the trihedron made by the straight lines which join the center of the Sun to three distant stars (if we want to study the motions of the solar system's planets).

Galilean (or inertial) frames

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But, as shown by Newton himself, that principe remains true for the relative motion of a free particle with respect to some particular reference frames, the Galilean frames.

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Galilean (or inertial) frames (2)

More exactly, let us assume that the principle of inertia holds true for the relative motion of free particles with respect to the reference frame defined by the rigid body R_1. What happens for the relative motion of these free particles with respect to another reference frame, defined by another rigid body R_2? It is easy to see that the principle of inertia still holds true if and only if the relative motion of $R - 2$ with respect to R_1 is a motion by translation at a constant speed.

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The absolute frame, if it exists, therefore appears as a Galilean frame among an infinite number of other Gallilean frames, that no measurement founded on mechanical properties can distinguish from the others. For this reason, several scientists, for example Leibniz, doubted about its existence was doubtful.

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Leibniz Space-Time, which will be denoted by $\mathcal{U}$ (for Universe), is the disjoint union of all the Spaces at time $t \in \mathcal{E}_t$, for all $t \in \mathcal{T}$. So, according to Leibniz views, we still have a Space-Time, but no more an absolute space !

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on the othe hand Leibniz Space-Time $\mathcal{U}$, endowed with only one natural projection onto absolute Time $\mathcal{T}$, still denoted by

$$p_2 : \mathcal{U} \to \mathcal{T}\,.$$

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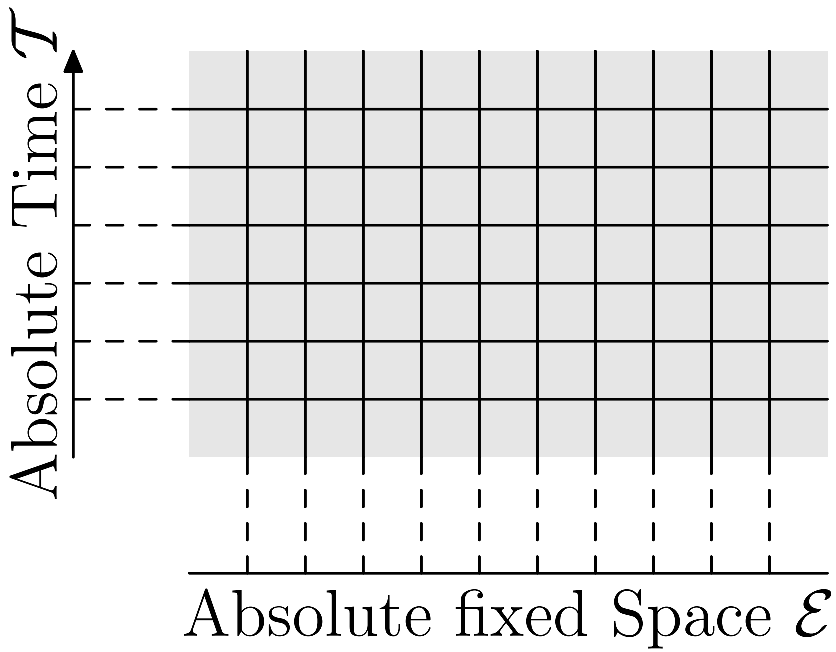
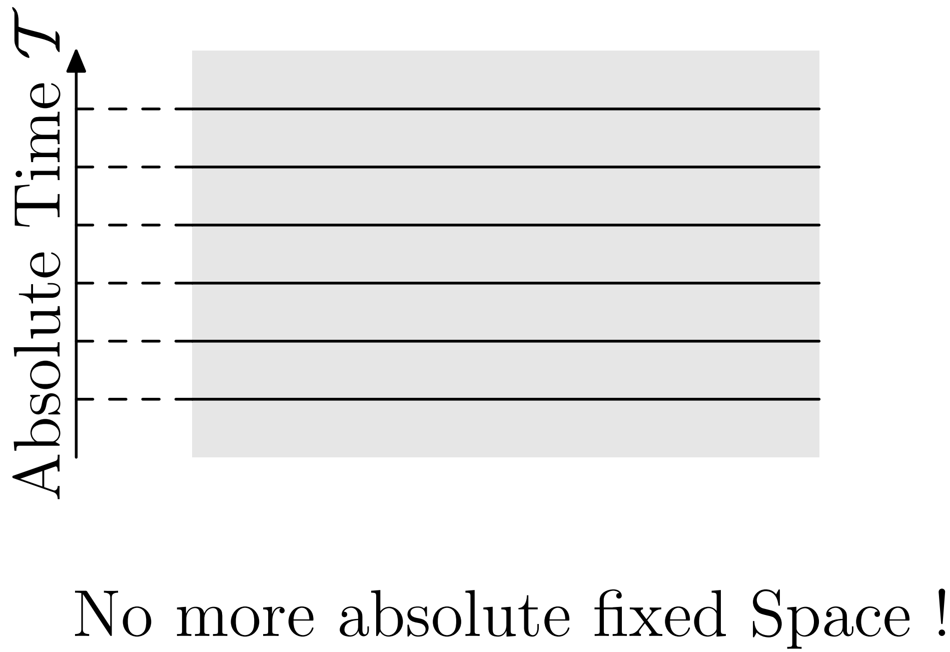


Newton's Space-Time



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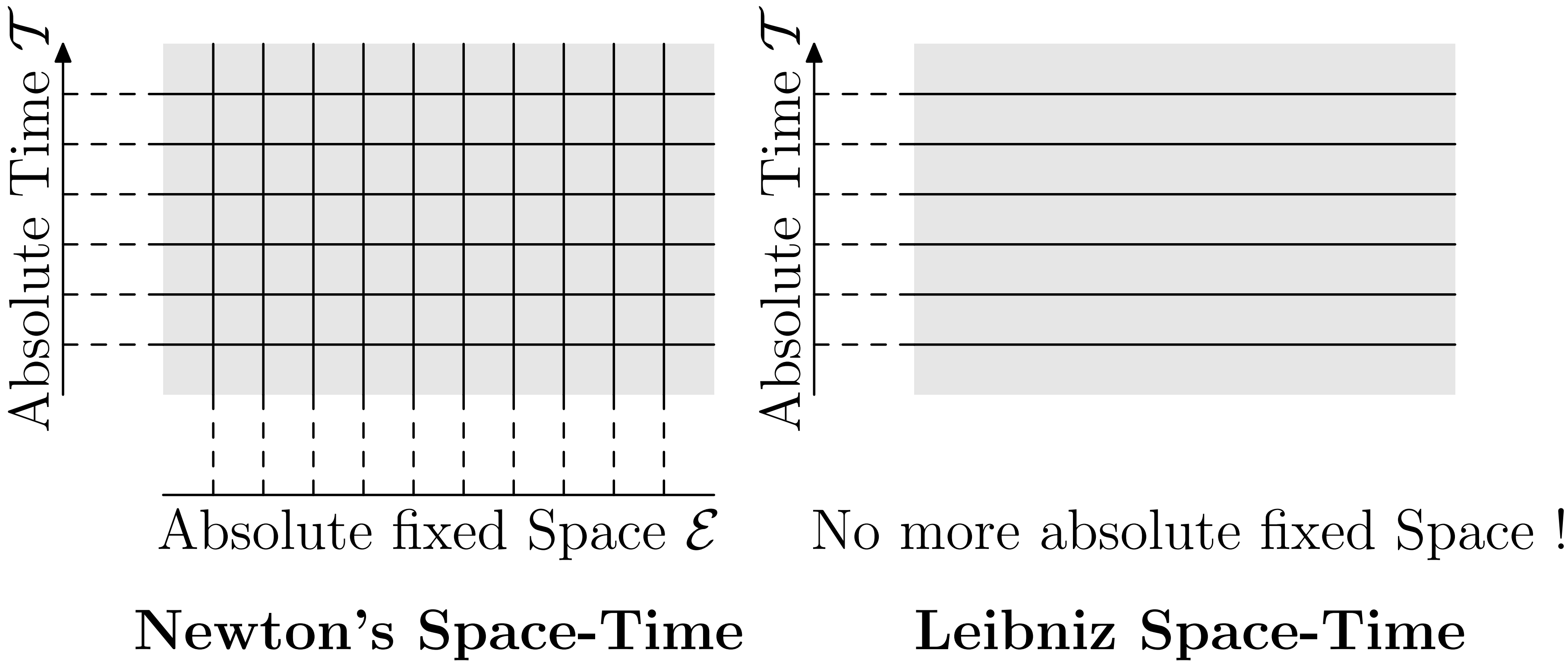

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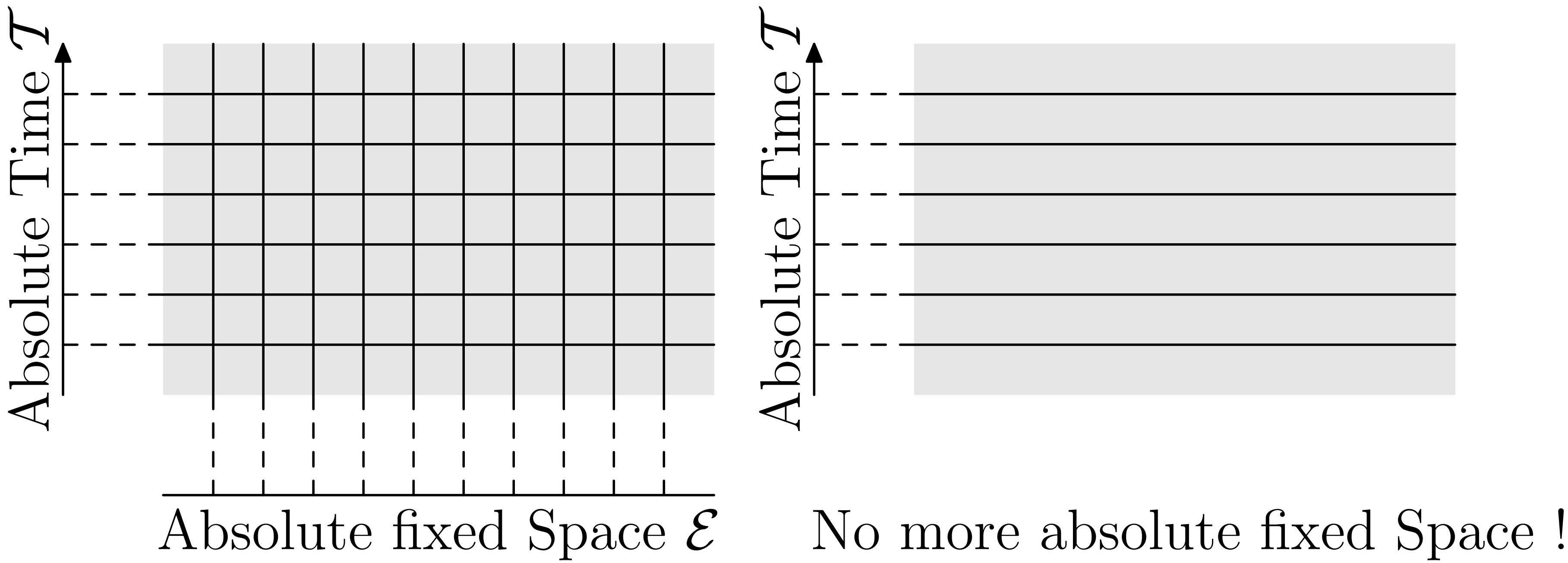

Newton's Space-Time **Leibniz Space-Time**

— But how do you put together the Spaces at various times $\mathcal{E}_t$ to make Leibniz Space-Time $\mathcal{U}$? Are they stacked in an arbitrary way?

— Of course no! The way in which they are stacked is not arbitrary, it is determined by the principle of inertia.

Leibniz Space-Time (4)

Leibniz Space-Time $\mathcal{U}$ is a 4-dimensional affine space, fibered (*via* an affine map) over Time $\mathcal{T}$, which is itself a 1-dimensional affine space. Its fibres, the Spaces $\mathcal{E}_t$ at various times $t \in \mathcal{T}$, are 3-dimensional Euclidean spaces. The affine structure of $\mathcal{U}$ is determined by the principle of inertia of which we have already spoken. That principle can be formulated in a way which does not use reference frames, by saying:

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So formulated, the principle of inertia can be applied to Newton's Space-Time $\mathcal{E} \times \mathcal{T}$ and to Leibniz Space-Time $\mathcal{U}$ as well! More, it determines the affine structure of $\mathcal{U}$, since one can easily show that the affine structure for which it holds true, if any, is unique. A physical law, the principle of inertia, is so embedded in the geometry of Leibniz Space-Time $\mathcal{U}$!

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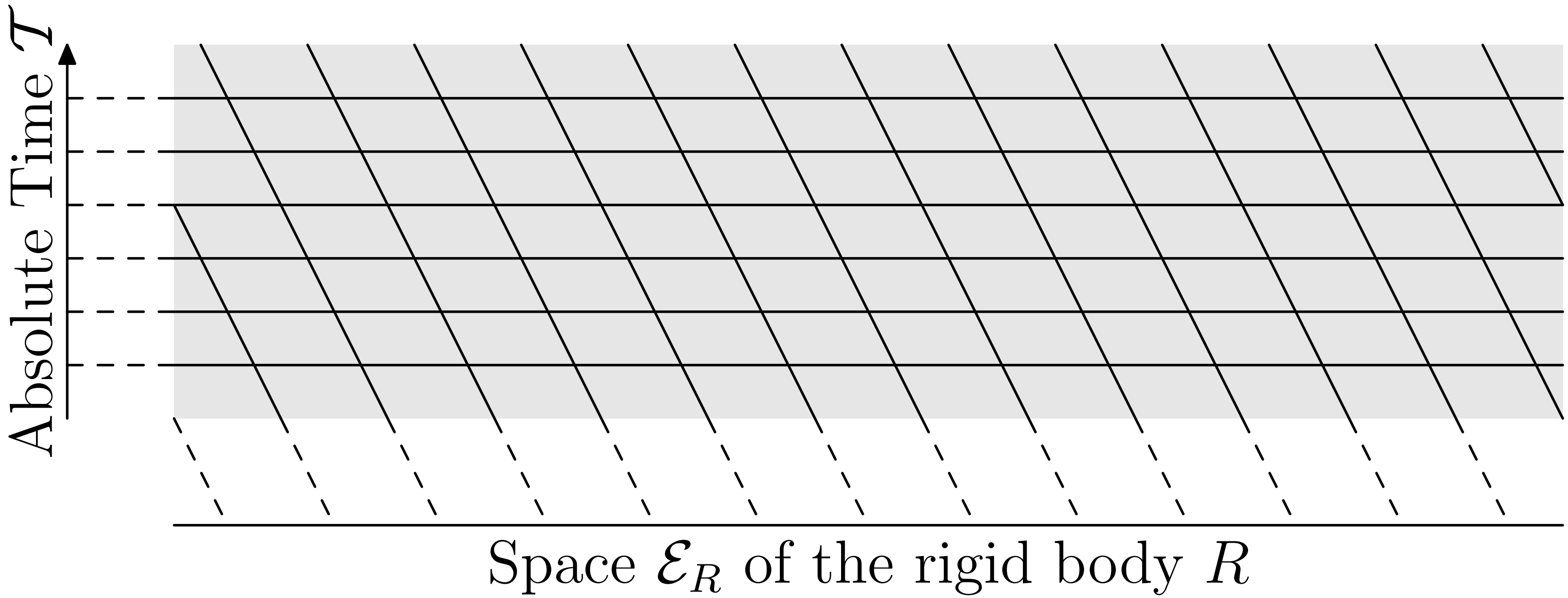

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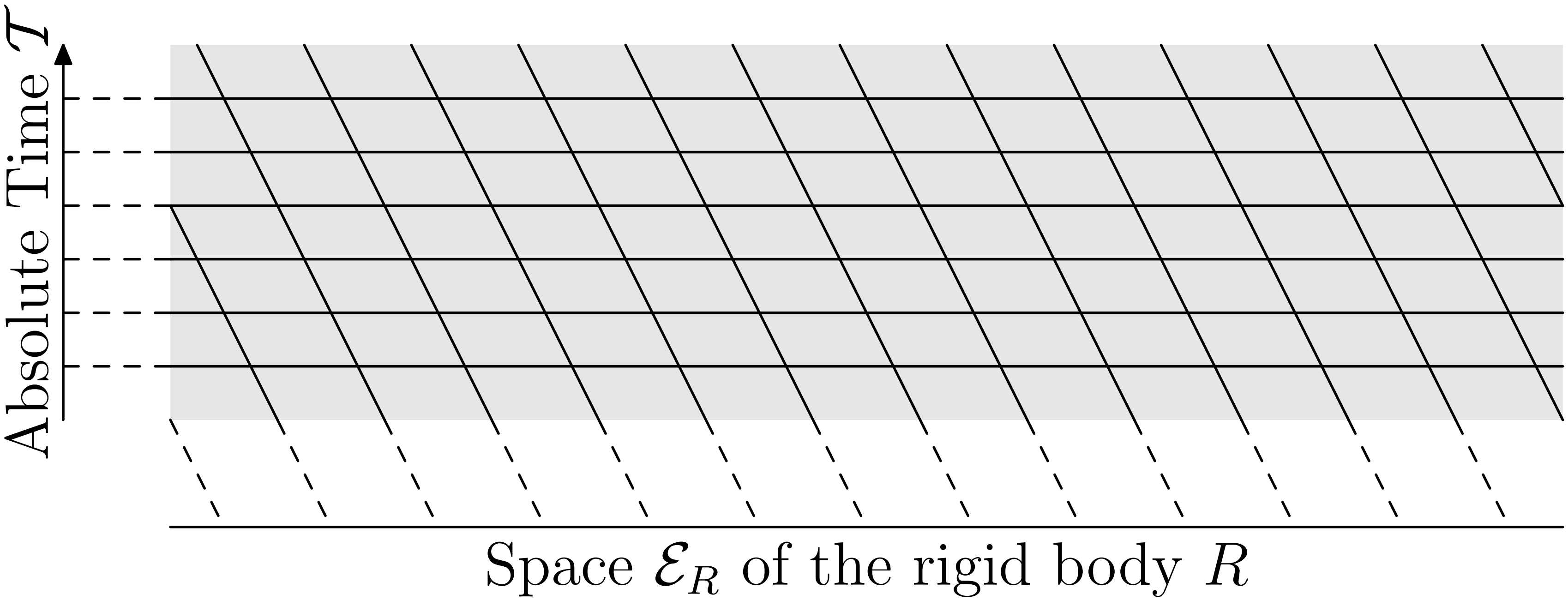

But of course, the space $\mathcal{E}_R$ depends on the choice of the reference frame $\mathcal{R}$.

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but according to Leibniz's views, when expressed as done above, it is Space-Time $\mathcal{U}$ which is directly related to reality, as well as absolute Time $\mathcal{T}$, and absolute Space $\mathcal{E}$ no more exists.

Light and Electromagnetism

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According to the theory built by the great Scotch physicist James Clerk Maxwell (1831–1879), electromagnetic phenomena propagate in vacuum as waves, with the same velocity in all directions, independently of the motion of the source of these phenomena. Maxwell soon understood that light was an elecromagnetic wave, and lots of experimental results confirmed his views.

The luminiferous ether

In Leibniz Space-Time (as well as in Newton's Space-Time) relative velocities behave additively. In that setting, it is with respect to at most one particular reference frame that light can propagate with the same velocity in all directions. Physicists introduced a new hypothesis: electromagnetic waves were considered as vibrations of an hypothetic, very subtle, but highly rigid medium called the luminiferous ether, everywhere present in space, even inside solid bodies. They thought that it was with respect to the ether's reference frame that light propagates at the same velocity in all directions. This new hypothesis amounts to come back to Newton's absolute Space identified with the ether. There were even physicists who introduced additional complications, by assuming that the ether, partially drawn by the motion of moving bodies, could deform with time!

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These results remained not understood until 1905, despite many attempts. The most interesting of these attempts was that due to Hendrik Anton Lorentz (1853–1928) and George Francis FitzGerald (1851–1901).

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— No! Not at all! Lorentz and FitzGerald considered that contraction as a true physical effect or the relative motion of a body with respect to the ether. The only positive effect of that hypothesis is that by thinking about it, Einstein discovered Special Relativity. But now, it is completely abandoned! The relativistic contraction of lengths and dilation of times has nothing to do with Lorentz and FitzGerald assumption: rather than a real phenomenon, it is only an appearance, like an effect of perspective.

No more absolute Time!

Einstein was the first [a] to understand (in 1905) that Michelson and Morley experiments could be explained by a deep change of the properties attributed to Space and Time. At that time, his idea appeared as truly revolutionary. But now it may appear as rather natural, if we think along the following lines:

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When we dropped Newton's Space-Time in favour of Leibniz Space-Time, we recognized that there is no absolute Space, but that Space depends on the choice of a reference frame.

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When we dropped Newton's Space-Time in favour of Leibniz Space-Time, we recognized that there is no absolute Space, but that Space depends on the choice of a reference frame. Maybe Time too is no more absolute than Space, and depends on the choice of a reference frame!

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He also kept the notion of a Galilean frame. In $\mathcal{M}$, a Galilean frame is determined by a direction of straight line. The bodies which are at rest with respect to that frame are those whose all points have their world lines parallel to that direction.

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— You said that a direction of straight line was enough to determine a Galilean frame. But how is that possible, since we no more have an absolute Time?

Light cones

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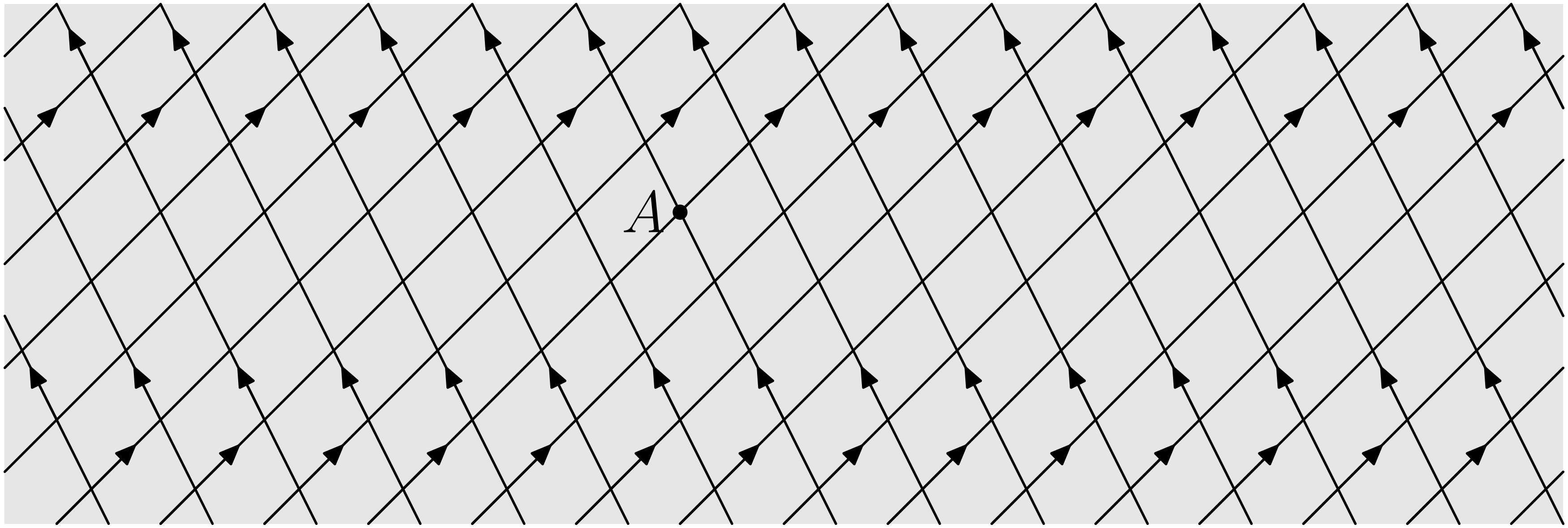
Given an event $A \in \mathcal{M}$, the light lines through A make a 3-dimensional cone, the light cone with apex A; the two layers of that cone are called the past half-cone and the future half-cone with apex A. The next picture shows schematically the light cones with various events as apexes, for a 2-dimensional Space-Time. Each of these cones is in that schematic picture, the union of two straight lines: the world line of a light signal going from left to right, and that of a light signal going from right to left.

Light cones (2)



Time-like and space-like straight lines

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Only directions of time-like straight lines determine a Galilean frame.

Isochronous subspaces

Let $\mathcal{A}$ be a time-like straight line. We want to determine the isochronous subspaces for the Galilean frame determined by the direction of $\mathcal{A}$. They must be such that the length covered by a light signal, calculated in that reference frame, during a given time interval, also evaluated in that reference frame, is the same in any two opposite directions.

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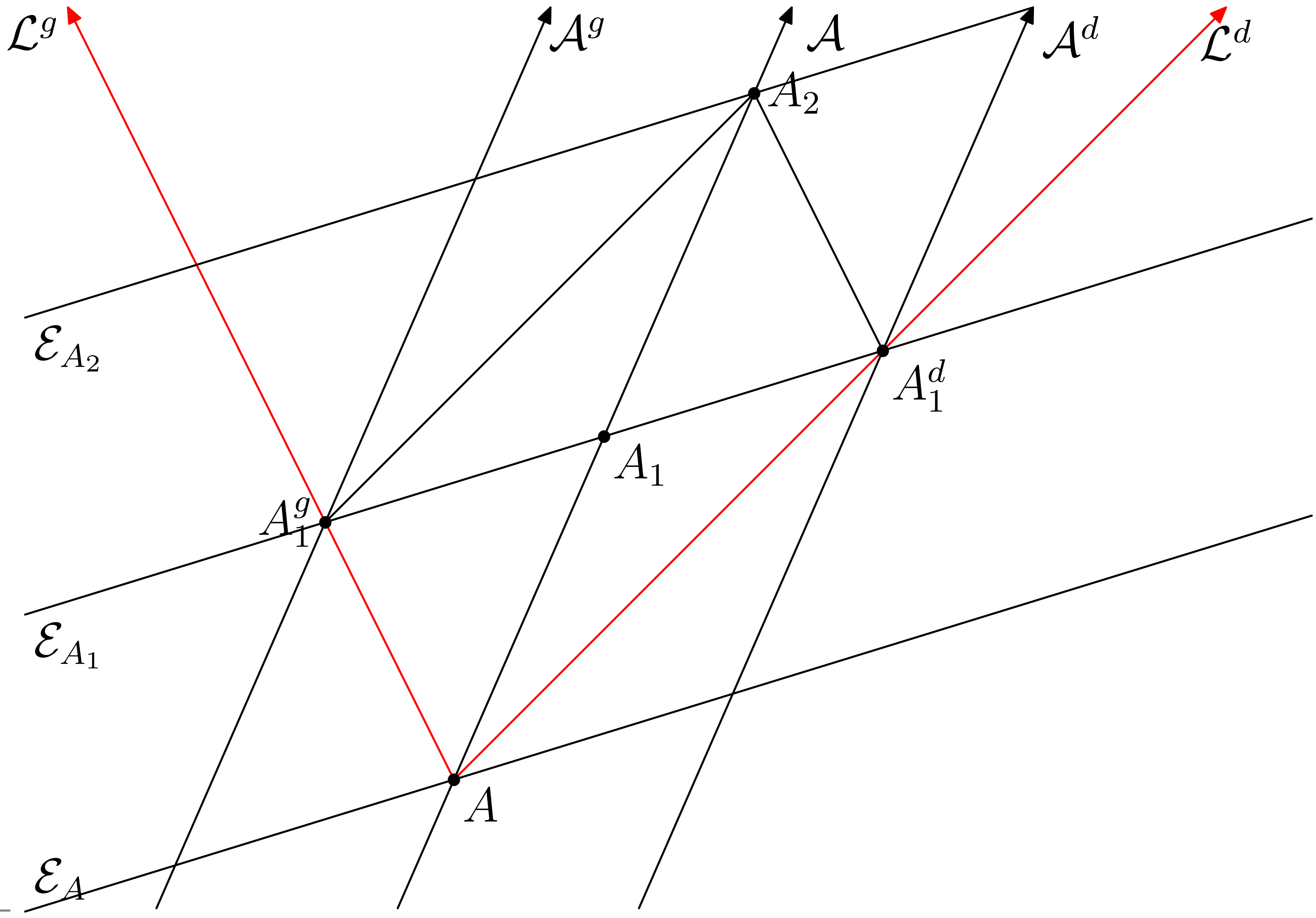
In a schematic 2-dimensional Space-Time, the direction of isochronous subspaces is easily obtained: we take the two light lines $\mathcal{L}^d$ and $\mathcal{L}^g$ through an event $A \in \mathcal{A}$; we take another event $A_1 \in A$, for example in the future of A, and we build the parallelogram with two sides supported by $\mathcal{L}^d$ and $\mathcal{L}^g$, with A as one of its apices and A_1 as center. The isochronous subspaces are all the straight lines parallel to the space-like diagonal of that parallelogram.

Isochronous subspaces (2)

A light signal starting from A covers, during the time interval between events A and A_1, the lengths $A\,A_1^g$ towards the left and $A\,A_1^d$ towards the right. These lengths are equal because $A_1^g\,A_1^d$ is the diagonal of a parallelogram whose center is A_1.

Isochronous subspaces (3)



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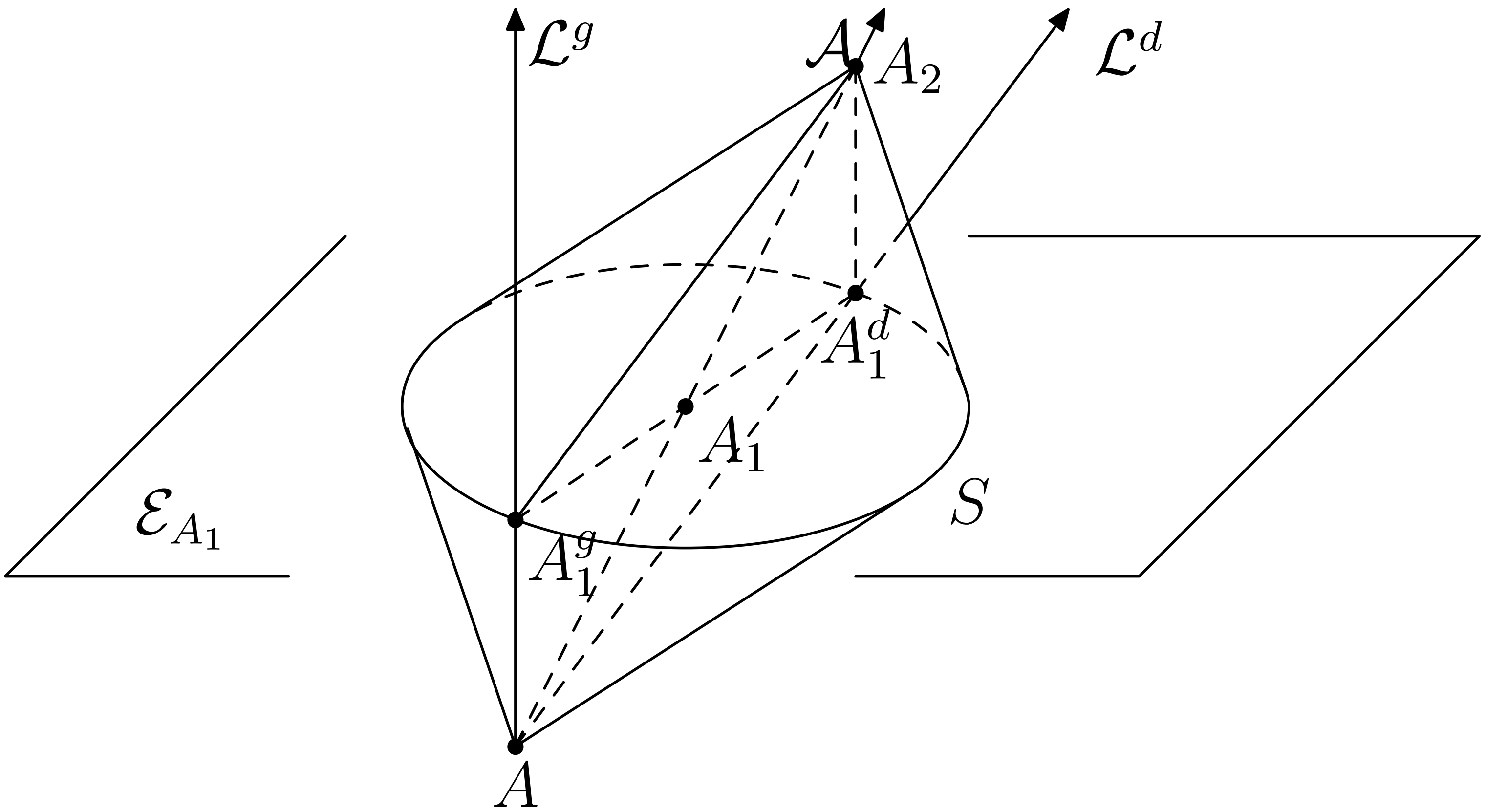
— It is the same. Take the event A_2 on the light line $\mathcal{A}$ such that A_1 is the middle point of $A\,A_2$, and consider two light half-cones: the future light half-cone with apex A, and the past light half-cone with apex A_2. Their intersection is a 2-dimensional sphere S_1. The unique affine hyperplane $\mathcal{E}_{A_1}$ which contains S_1 is an isochronous subspace for the Galilean frame determined by the direction of $\mathcal{A}$. The other isochronous subspaces for that Galilean frame are all the hyperplanes parallel to $\mathcal{E}_{A_1}$.

Isochronous subspaces (5)



Change of Galilean frame

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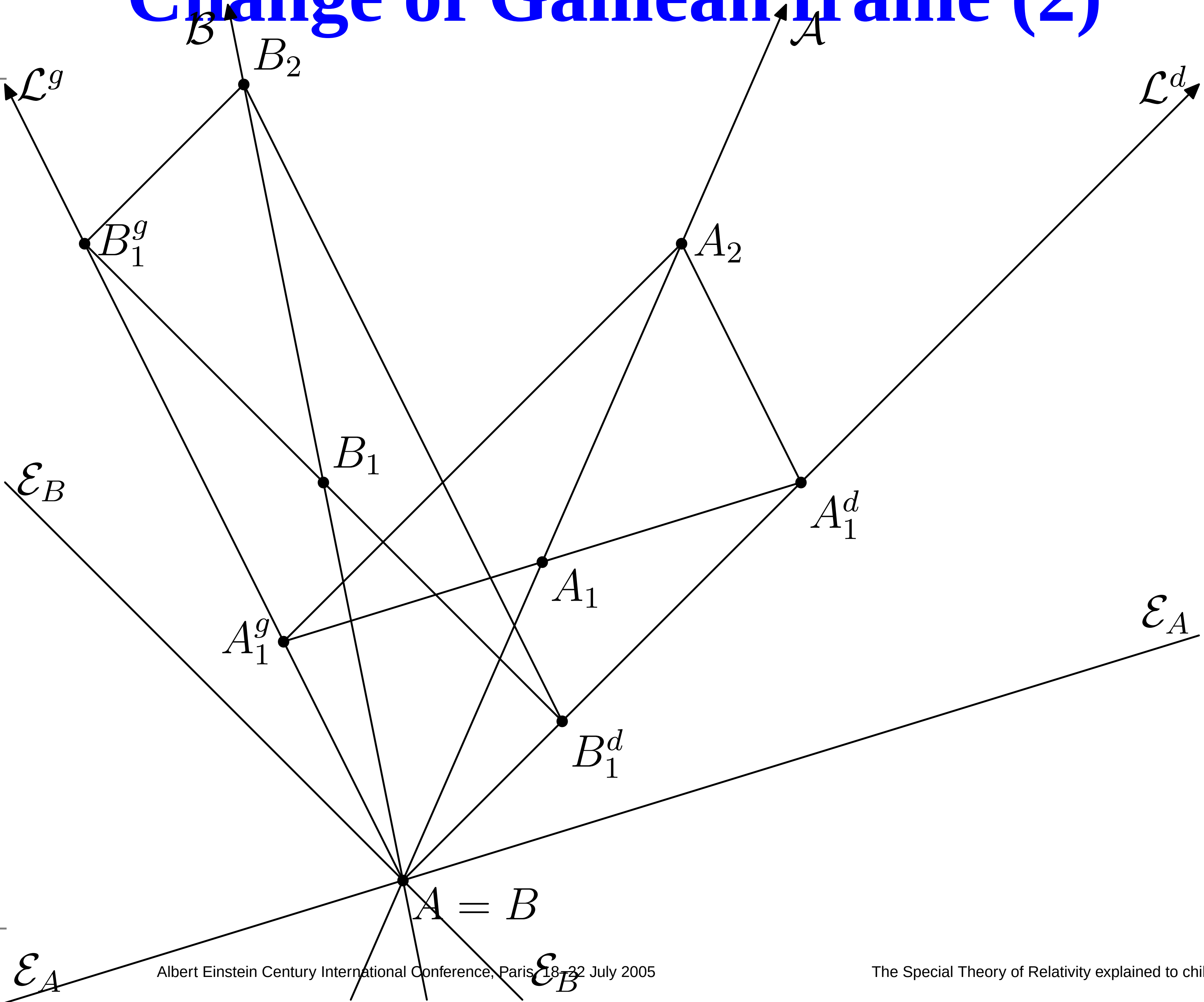
Moreover, contrary to what happened in Leibniz Space-Time, the direction isochronous subspaces is also changed! Therefore, the chronological order of two events can be different when it is appreciated in two different Galilean frames!

Change of Galilean frame (2)



Comparison of time intervals

Up to now, we have compared the lengths of two straight line segments in $\mathcal{M}$ only when they were supported by parallel straight line. That was allowed by the affine structure of $\mathcal{M}$.

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We need more, because the spectral lines of atoms allow us to build clocks and to compare time intervals measured in two different Galilean frames.

Comparison of time intervals (2)

Let $A\,A_1$ and $A\,B_1$ be two straight line segments supported by two different time-like straight lines $\mathcal{A}$ and $\mathcal{B}$, which meet at the event A. How can we assert that the time intervals corresponding to $A\,A_1$ measured in the Galilean frame determined by the direction of $\mathcal{A}$, and to $A\,B_1$ measured in the Galilean frame determined by the direction of $\mathcal{B}$, are the same?

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Answer: these two time intervals are equal if and only if the events A_1 and B_1 lie on the same arc of hyperbola which has the light lines $\mathcal{L}^d$ and $\mathcal{L}^g$ (which meet at A and are contained in the two-dimensional plane which contains $\mathcal{A}$ and $\mathcal{B}$) as asymptotes. Or more generally, on the same hyperboloid with the light cone of A as asymptotic cone.

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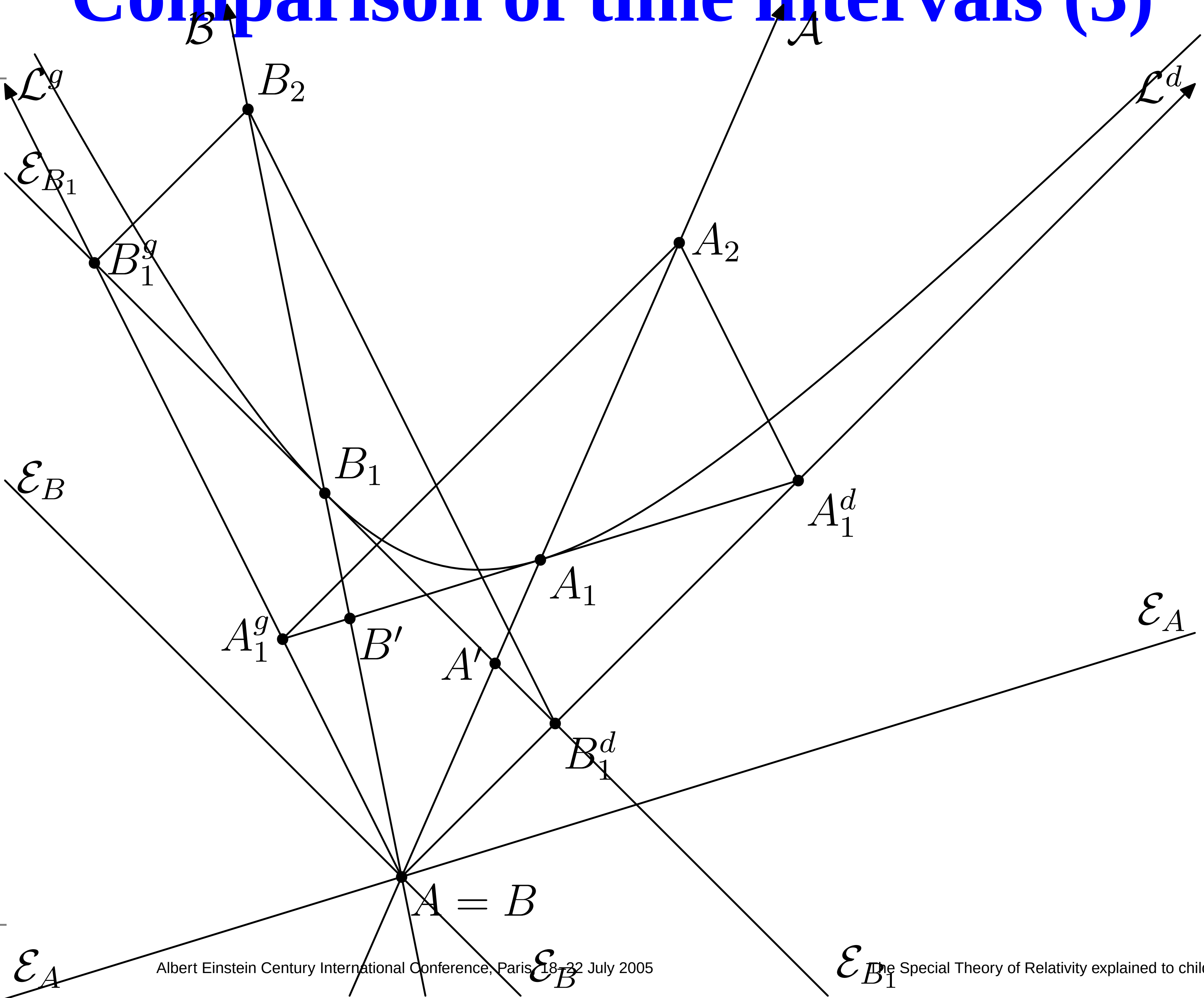

Comparison of lengths

The comparison of lengths on two non-parallel space-like straight lines follows from the comparison of time intervals: the natural measure of a segment on a space-like straight line is the time taken by a light signal to cover that length, measured in a Galilean frame in which that segment lays in an isochronous subspace.

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Let $A\,A^d$ and $A\,B^d$ be two straight line segments supported by two space-like straight lines $\mathcal{E}_A$ et $\mathcal{E}_B$, which meet at the event A. They are of equal length if and only if A^d and B^d lie on the same hyperboloid with the light cone of A as asymptotic cone.

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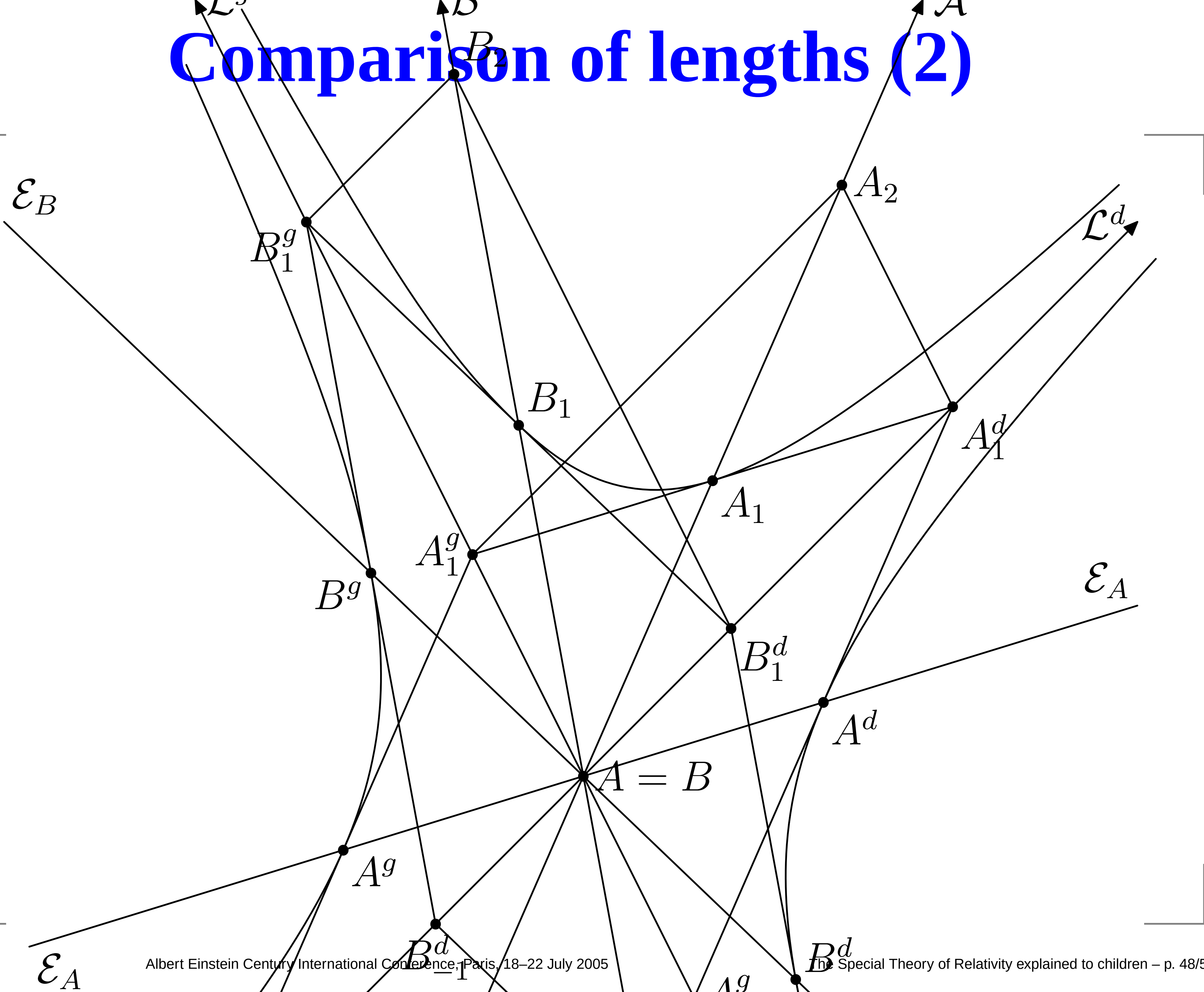

Conclusion

The comparison of time intervals and lengths presented above, founded on very simple geometric arguments, allows a very natural introduction of the pseudo-Euclidean metric of Minkowski's Space-Time. The construction of isochronous subspaces in two different Galilean frames, as presented above, leads to the formulas for Lorentz transformations with a minimum of calculations.

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The pictures we have presented allow a very easy explanation of the apparent contraction of lengths and dilation of times associated to a change of Galilean frame.

Conclusion (2)

Given two events, with one in the future of the other, the straight line is the longest time-like path (when measured in proper time) which joins these two events. That property leads to a very simple explanation, without complicated calculations, of the (improperly called) paradox of Langevin's twins.

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By explaining that the affine structure of Space-Time should be questioned, a smooth transition towards General Relativity, suitable from children 8 to 108 years old, seems possible.