The Special Theory of Relativity explained to children

(from 7 to 107 years old)

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Prologue

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This is the discussion I would like to have with her (or him).

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— Time and space seem to me very intuitive, and yet difficult to understand in deep . . .

Prologue (3)

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Let me now indicate how the mental pictures of Time and Space used by scientists have evolved, mainly from Newton to Einstein.

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Newton considered as possible to compare two time intervals, even when they were many centuries apart. In modern mathematical language, that property determines an affine structure on $\mathcal{T}$.

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Other important property of Time: it always flows from past to future. To take it into account, we endow $\mathcal{T}$ with an orientation; it means that we consider the two directions (from past to future and from future to past) as not equivalent.

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For Newton, every object A of the physical world occupies, at each time t (element of $\mathcal{T}$) for which that object exists, a position A_t in Space $\mathcal{E}$. The motion of of A is described by its successive positions A_t when t varies in $\mathcal{T}$.

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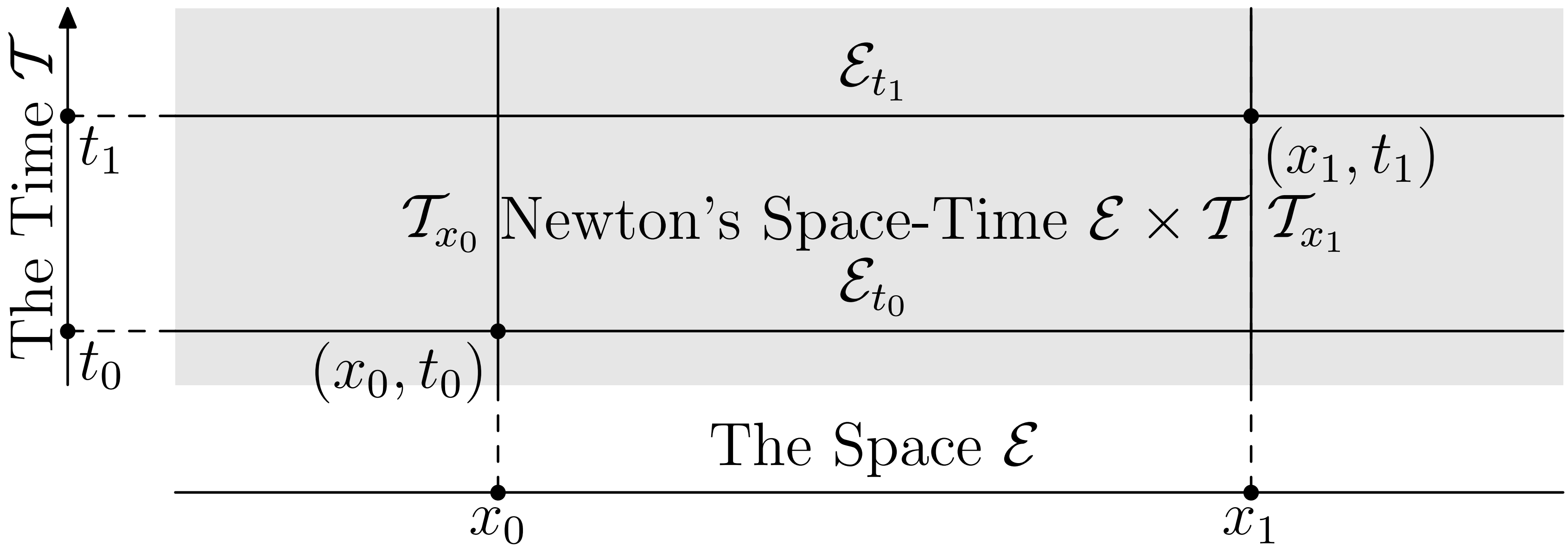

Newton's Space-Time (2)

Space-Time is very convenient to describe motions. For example, the motion of a material particle A is described by a line in $\mathcal{E} \times \mathcal{T}$, made by the events (A_t, t), for all t in the interval of time during which A exists. That line is called the world line of A.

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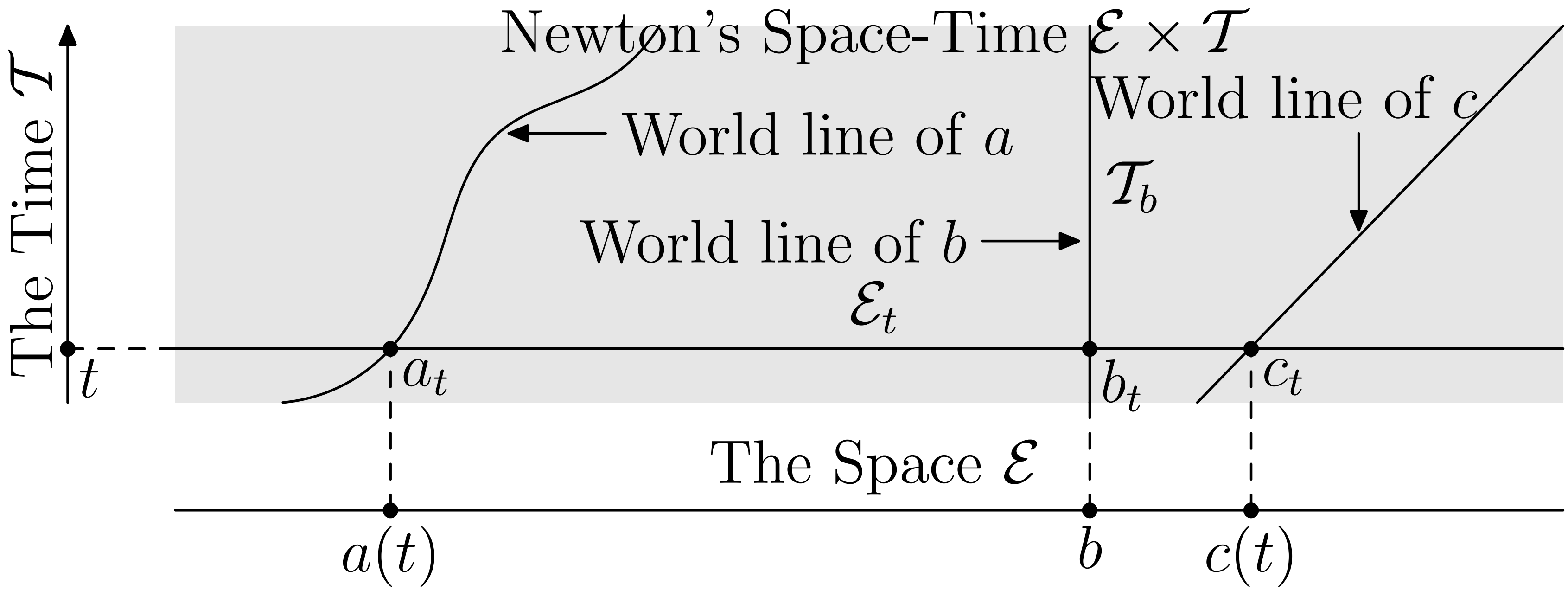

Absolute rest and motion

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— Because nothing is at rest in the Universe! The Earth rotates around its axis and around the Sun, which rotates around the center of our Galaxy. And there are billions of galaxies in the Universe, each one moving with respect to any other one. For these reasons, Newton's concept of an absolute Space was criticized very early, notably by his contemporary, the great mathematician and philosopher Gottfried Wilhelm Leibniz (1647–1716).

Reference frames

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— He used a reference frame. It means that he used a body R which remained approximately rigid during the motion he wanted to study, and he made as if that body was at rest. Then he could study the relative motion of any moving body A with respect to R.

Galilean (or inertial) frames

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That principe remains true for the relative motion of a free particle with respect to all reference frames determined by rigid bodies whose absolute motion is a translation at a constant velocity. These reference frames are the Galilean frames. No measurement founded on mechanical properties can distinguish the absolute frame from another Galilean frame.

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Leibniz Space-Time, which will be denoted by $\mathcal{U}$ (for Universe), is the disjoint union of all the Spaces at time $t \in \mathcal{E}_t$, for all $t \in \mathcal{T}$. So, according to Leibniz views, we still have a Space-Time, but no more an absolute space !

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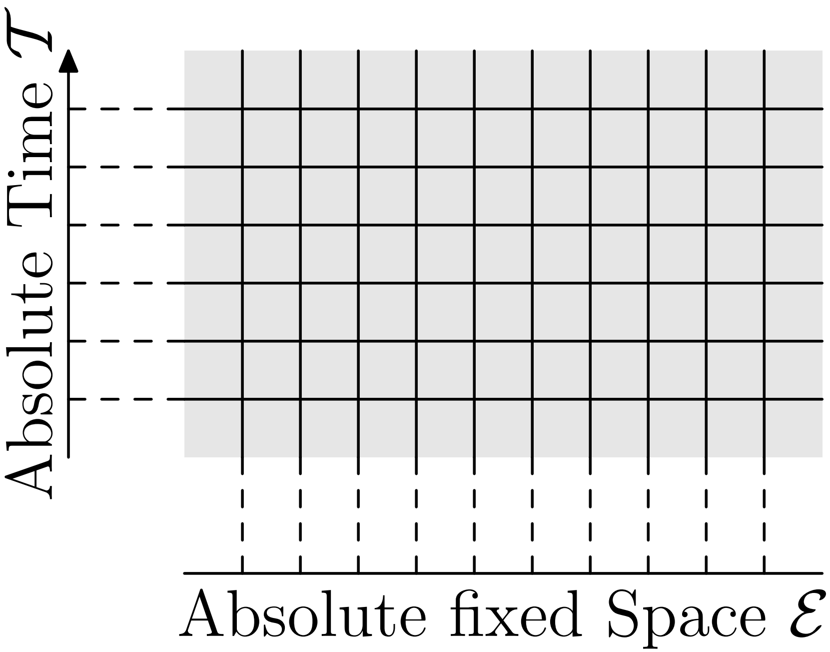

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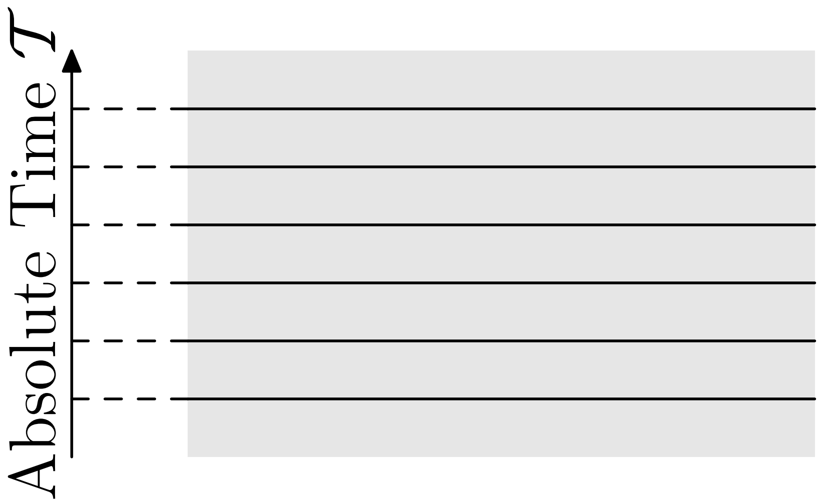

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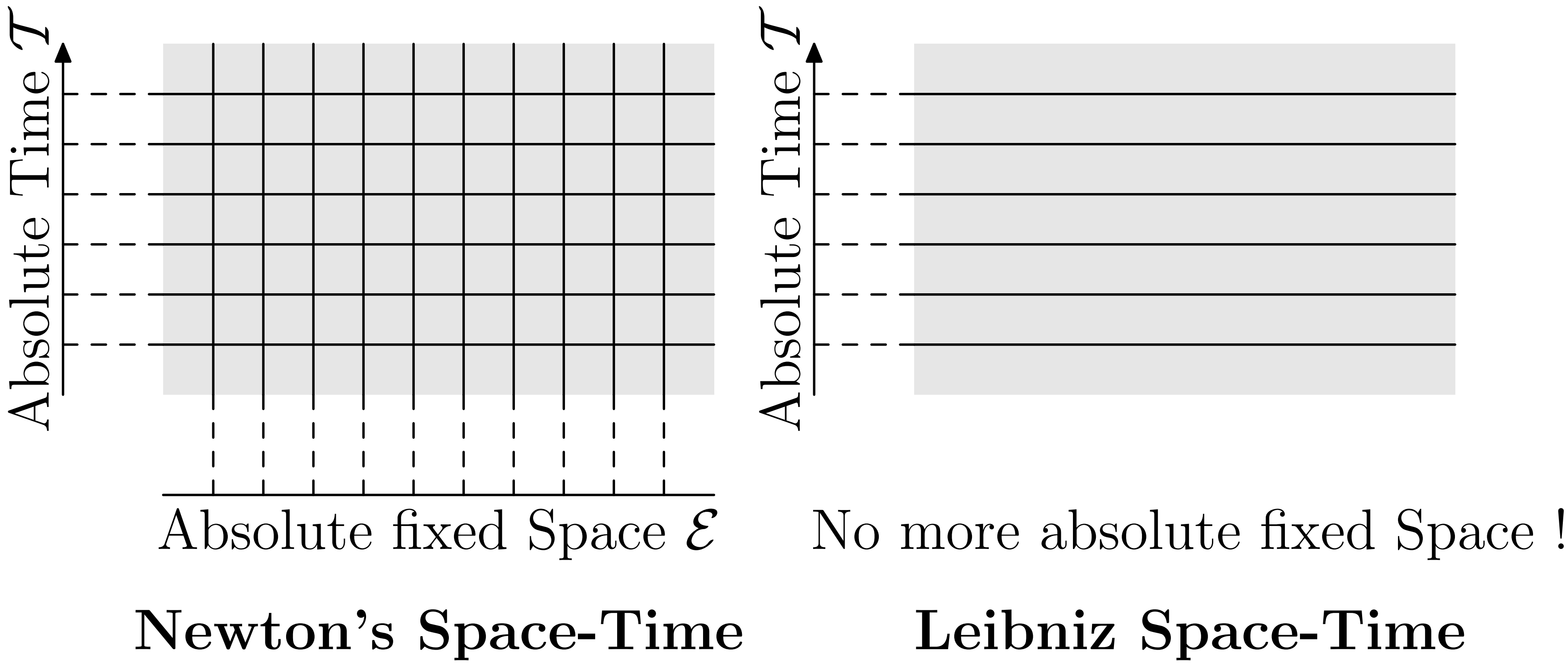

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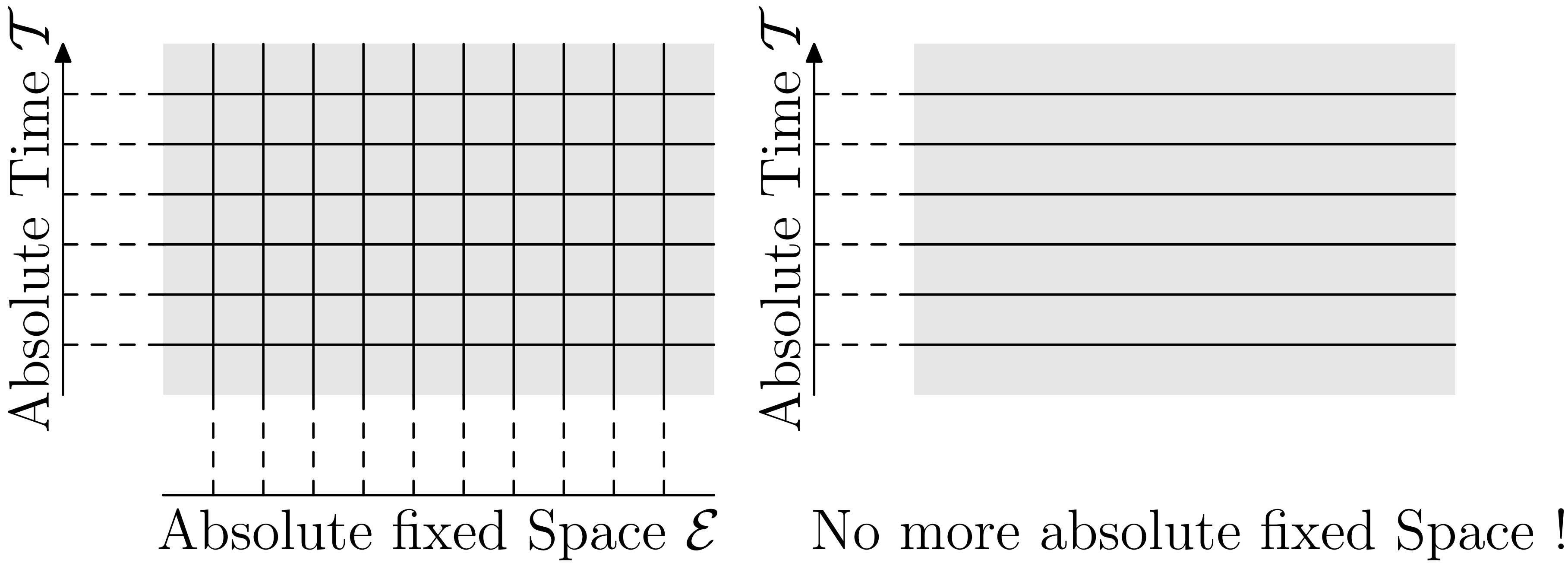

Newton's Space-Time **Leibniz Space-Time**

— But how do you put together the Spaces at various times $\mathcal{E}_t$ to make Leibniz Space-Time $\mathcal{U}$? Are they stacked in an arbitrary way?

— Of course no! The way in which they are stacked is not arbitrary, it is determined by the principle of inertia.

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So formulated, the principle of inertia determines the affine structure of $\mathcal{U}$. A physical law, the principle of inertia, is so embedded in the geometry of Leibniz Space-Time $\mathcal{U}$!

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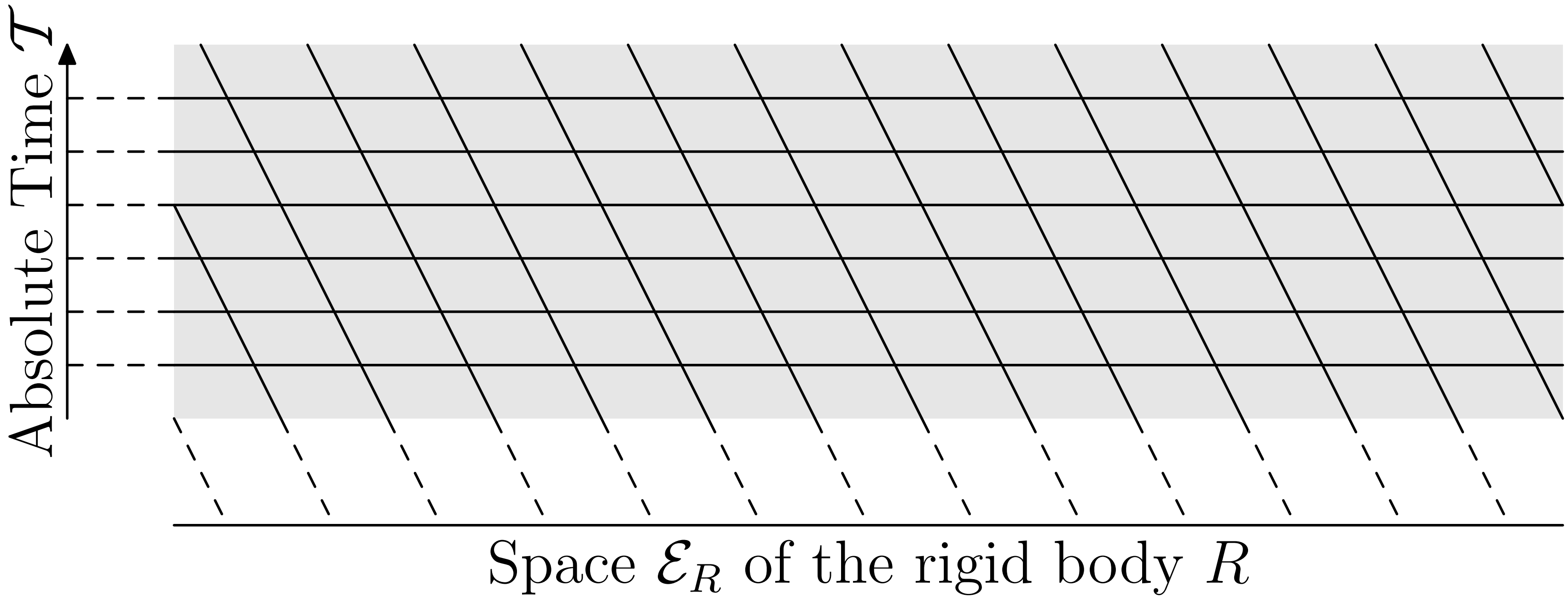

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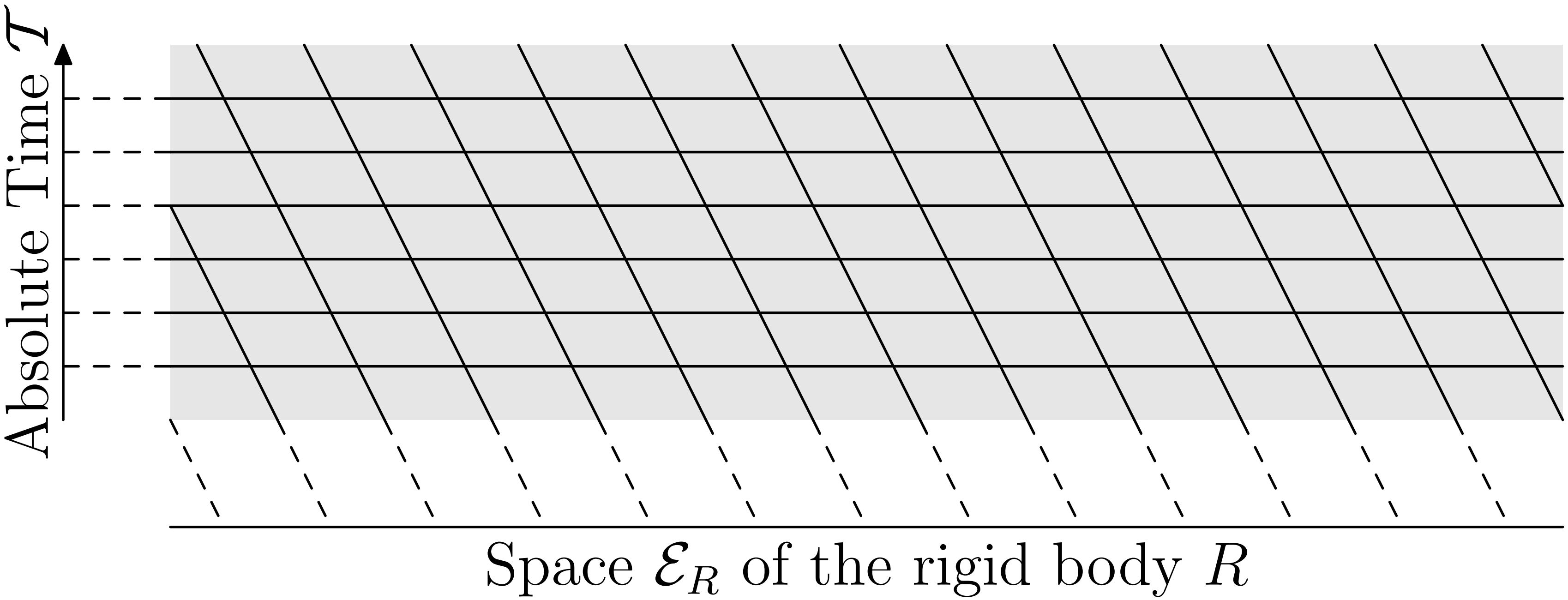

But of course, the space $\mathcal{E}_R$ depends on the choice of the reference frame $\mathcal{R}$.

Light and Electromagnetism

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According to the theory built by the great Scotch physicist James Clerk Maxwell (1831–1879), electromagnetic phenomena propagate in vacuum as waves, with the same velocity in all directions, independently of the motion of the source of these phenomena. Maxwell soon understood that light was an elecromagnetic wave, and lots of experimental results confirmed his views.

The luminiferous ether

In Leibniz Space-Time (as well as in Newton's Space-Time) relative velocities behave additively. In that setting, it is with respect to at most one particular reference frame that light can propagate with the same velocity in all directions. Physicists introduced a new hypothesis: electromagnetic waves were considered as vibrations of an hypothetic, very subtle, but highly rigid medium called the luminiferous ether. They thought that it was with respect to the ether's reference frame that light propagates at the same velocity in all directions. This amounts to come back to Newton's absolute Space identified with the ether. There were even physicists who introduced additional complications, by assuming that the ether, partially drawn by the motion of moving bodies, could deform with time!

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These results remained not understood until 1905, despite many attempts.

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He also kept the notion of a Galilean frame. In $\mathcal{M}$, a Galilean frame is determined by a direction of straight line. The bodies which are at rest with respect to that frame are those whose all points have their world lines parallel to that direction.

Light cones

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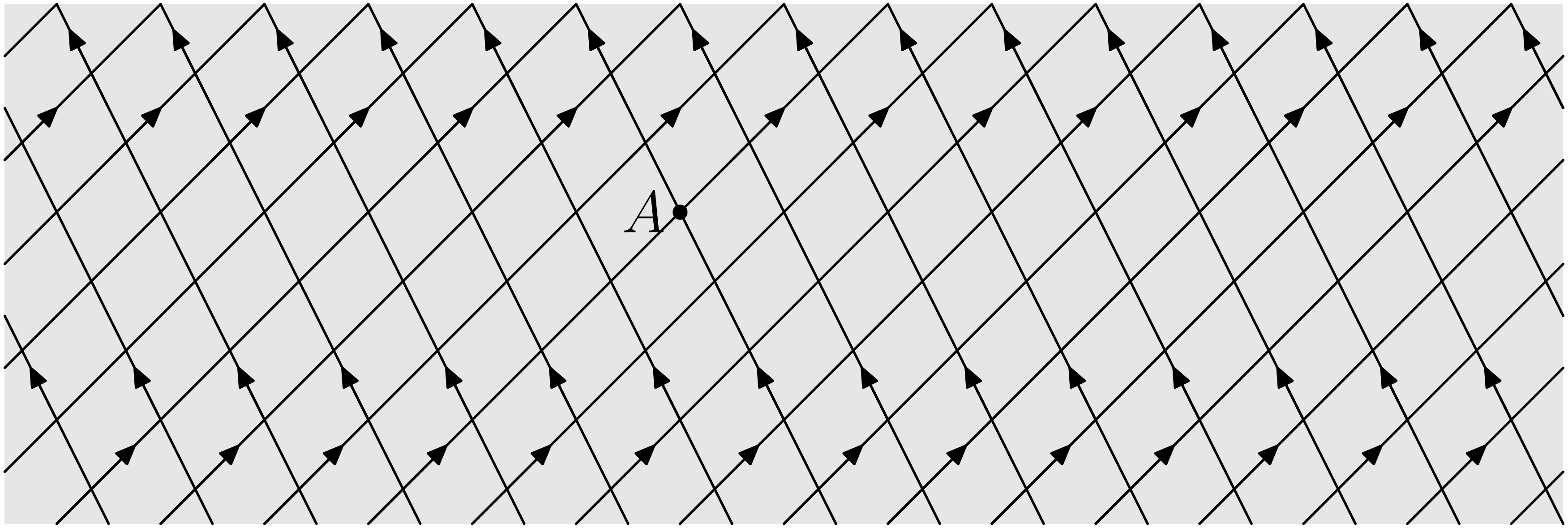
Given an event $A \in \mathcal{M}$, the light lines through A make a 3-dimensional cone, the light cone with apex A; the two layers of that cone are called the past half-cone and the future half-cone with apex A.

Light cones, time-like and space-like lines



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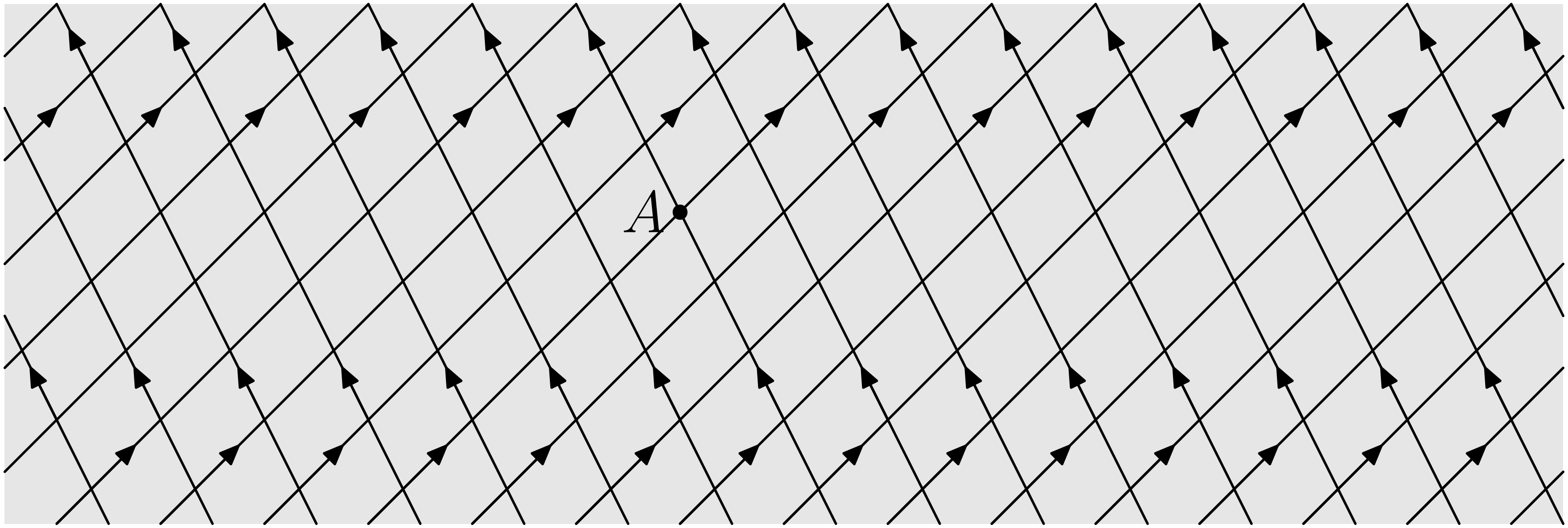

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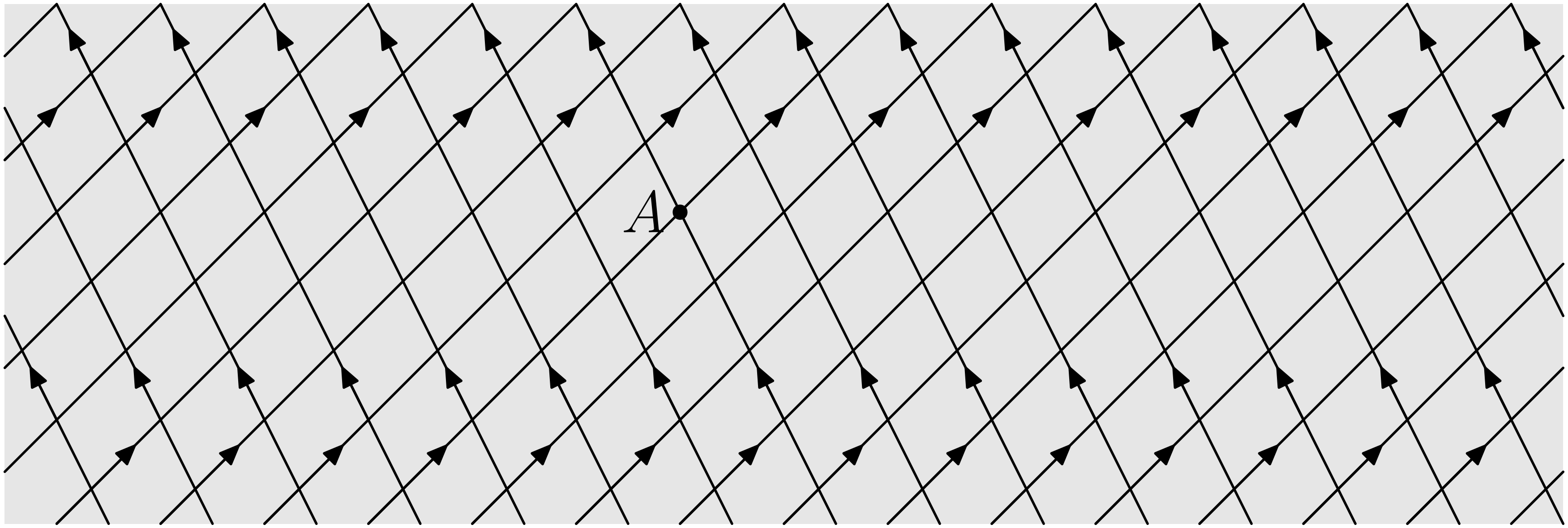

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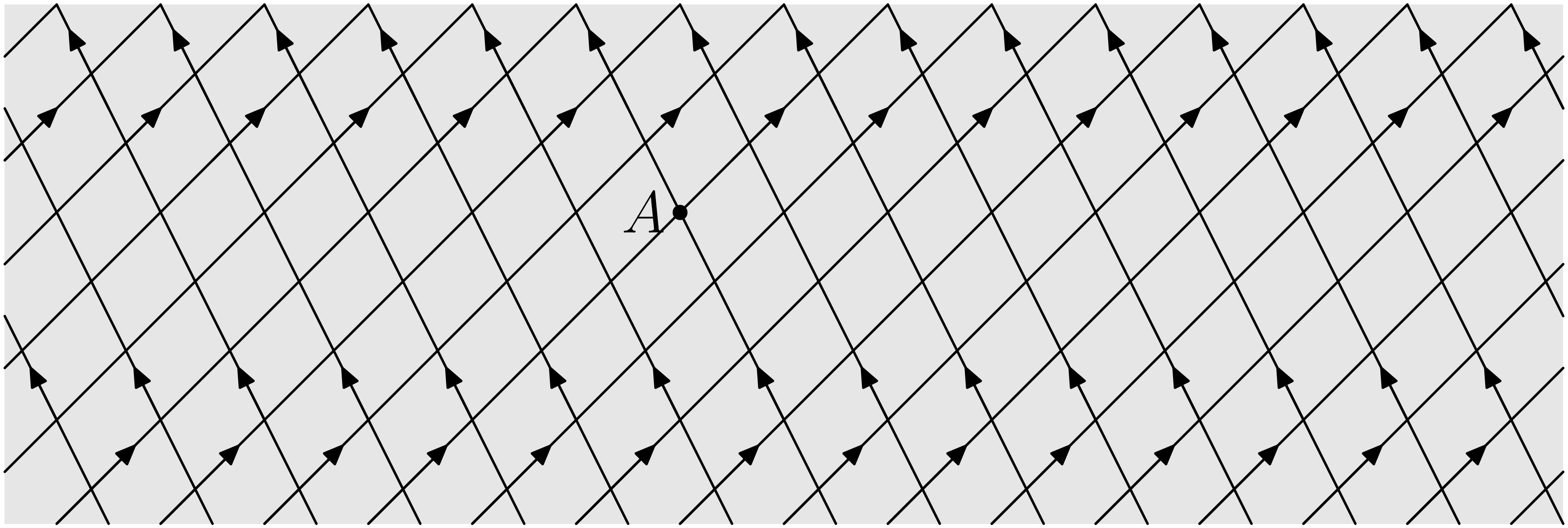

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Only directions of time-like straight lines determine a Galilean frame.

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— You said that a direction of straight line was enough to determine a Galilean frame. But how is that possible, since we no more have an absolute Time?

Isochronous subspaces

Let $\mathcal{A}$ be a time-like straight line. We want to determine the isochronous subspaces for the Galilean frame determined by the direction of $\mathcal{A}$. They must be such that the length covered by a light signal, calculated in that reference frame, during a given time interval, also evaluated in that reference frame, is the same in any two opposite directions.

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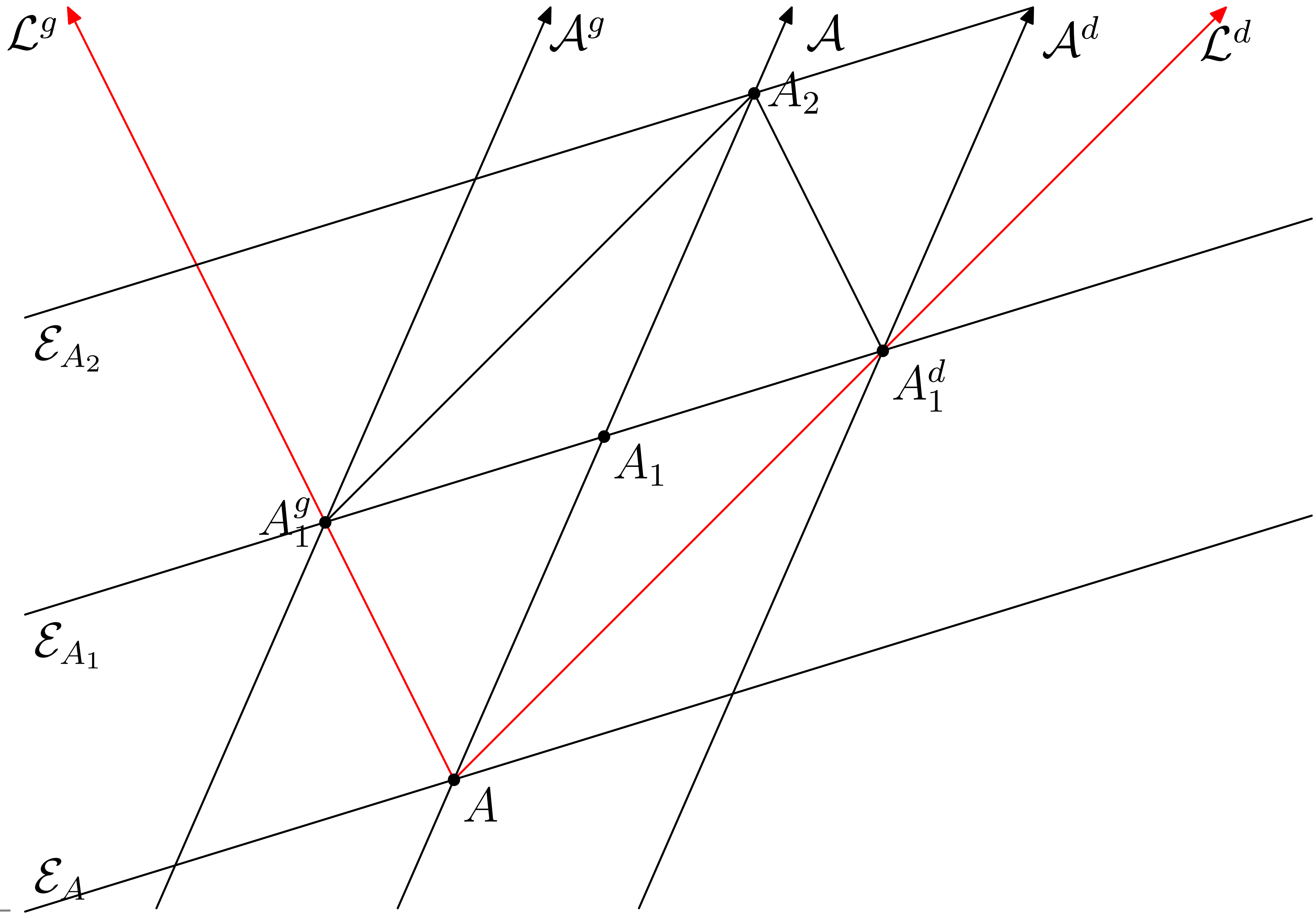
In a schematic 2-dimensional Space-Time, the direction of isochronous subspaces is easily obtained as shown on the following picture, by using the theorem which says that the diagonals of a parallelogram meet at their middle point!

Isochronous subspaces (2)



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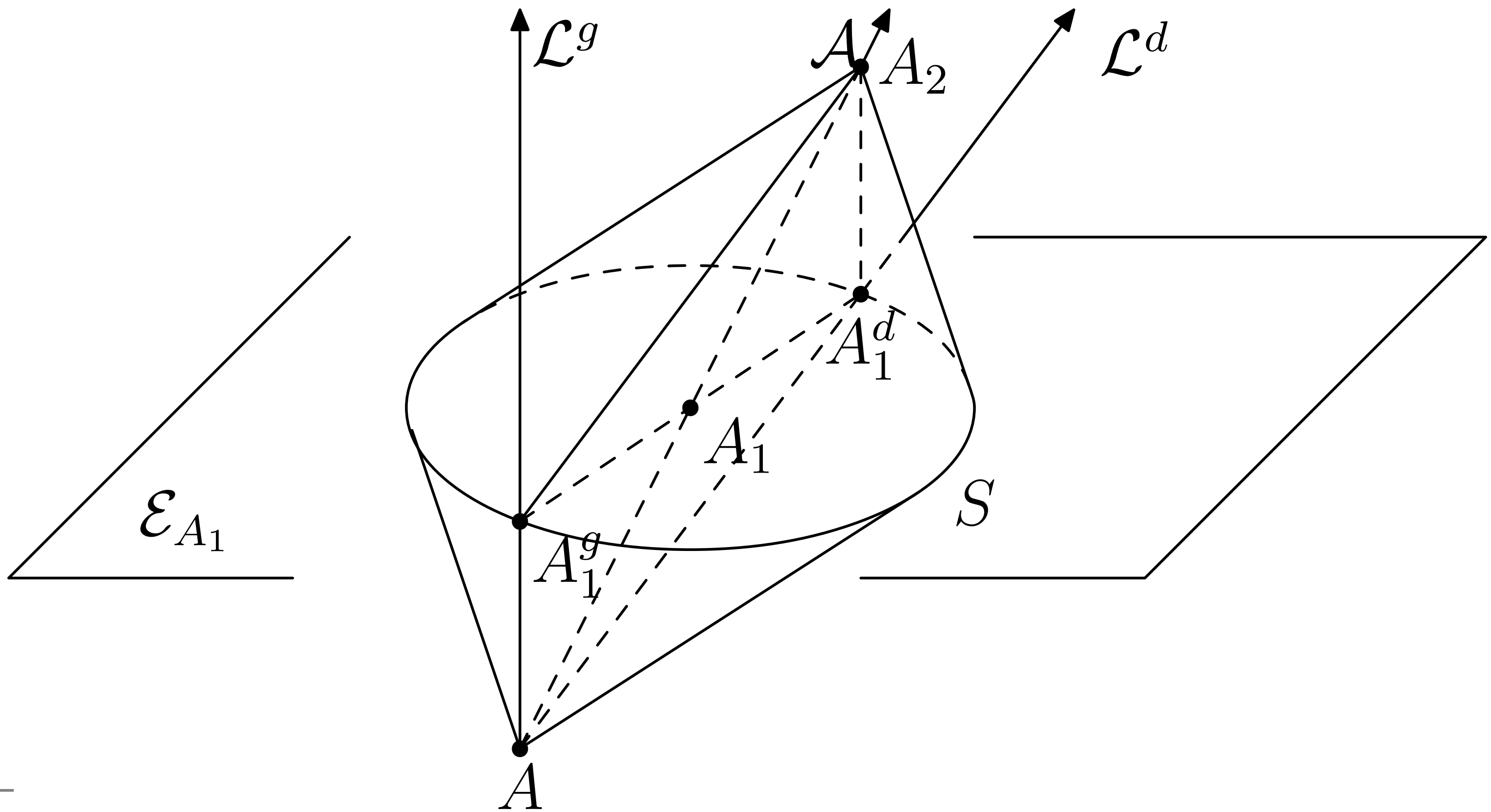

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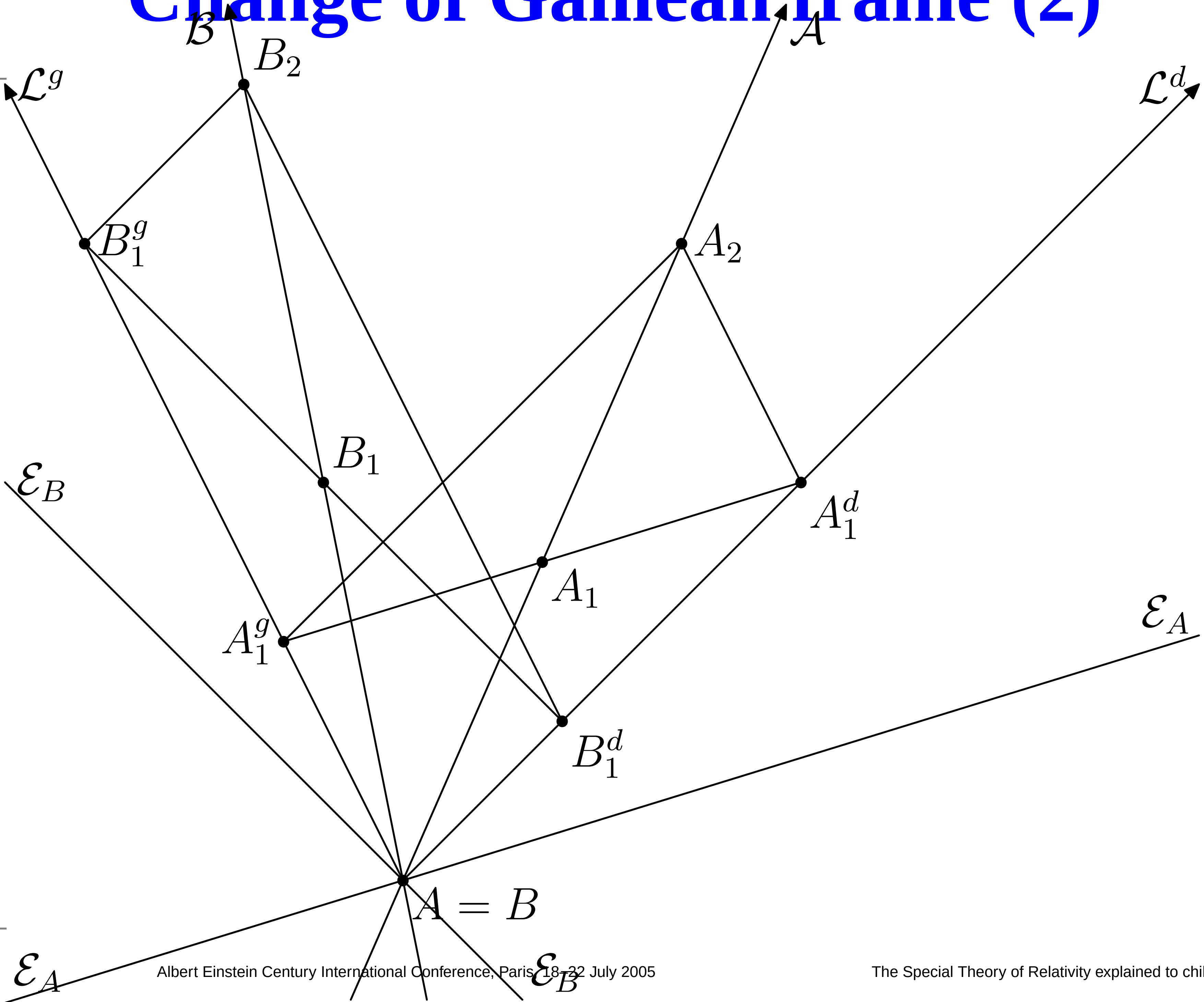
Moreover, contrary to what happened in Leibniz Space-Time, the direction isochronous subspaces is also changed! Therefore, the chronological order of two events can be different when it is appreciated in two different Galilean frames!

Change of Galilean frame (2)



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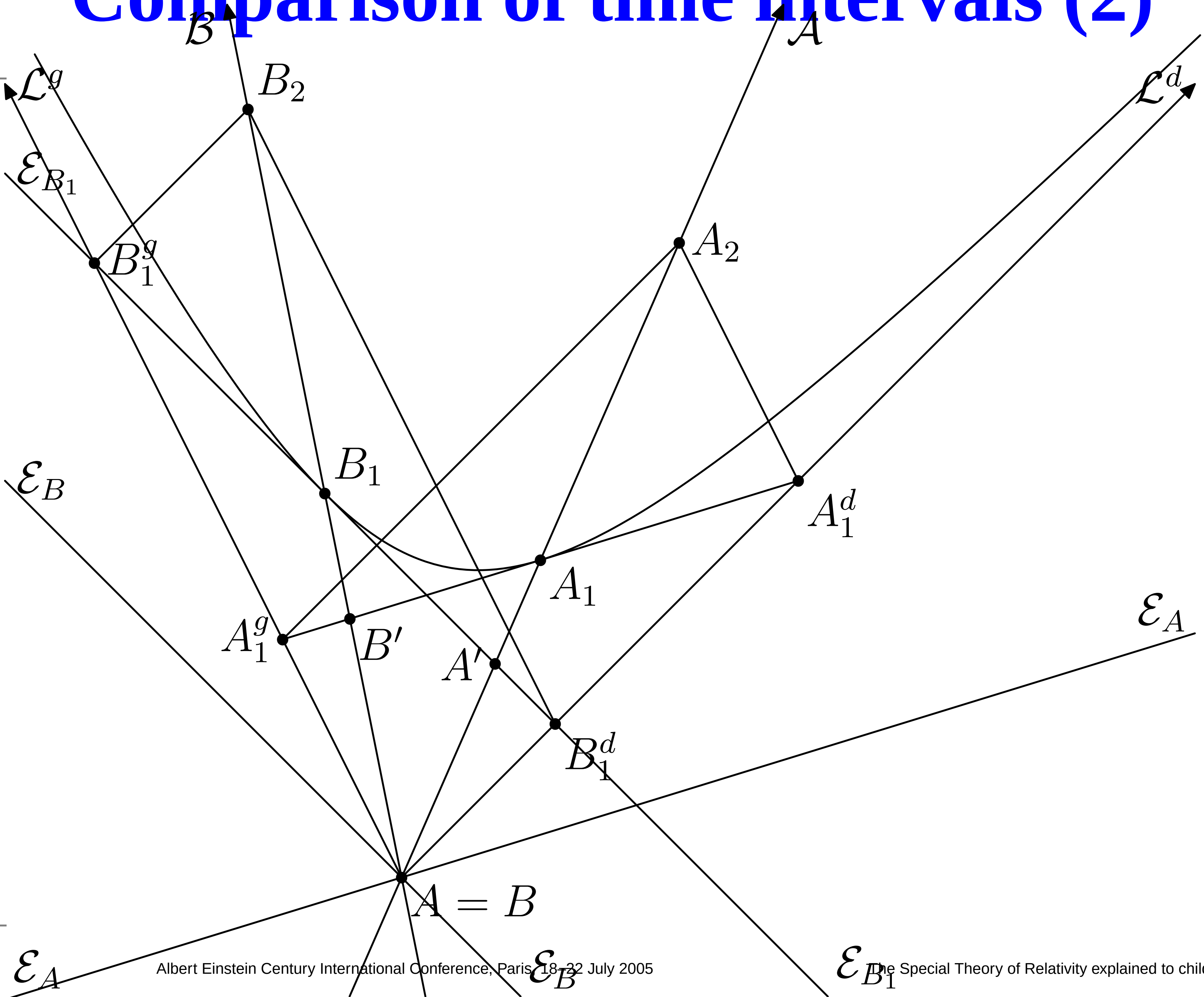
Two line segments $A\,A_1$ and $A\,B_1$ supported by two different time-like straight lines $\mathcal{A}$ and $\mathcal{B}$ are equal (when each one is measured in its own Galilean frame) if and only if the events A_1 and B_1 lie on the same arc of hyperbola which has the light lines $\mathcal{L}^d$ and $\mathcal{L}^g$ as asymptotes.

Comparison of time intervals (2)



Comparison of time intervals (3)

That follows from the equality, which says that the apparent dilation of times is the same for each Galilean frame with respect to the other,

$$\frac{A\,A_1}{A\,A'} = \frac{A\,B_1}{A\,B'}\,.$$

Comparison of lengths

The comparison of lengths on two non-parallel space-like straight lines follows from the comparison of time intervals: the natural measure of a segment on a space-like straight line is the time taken by a light signal to cover that length, measured in a Galilean frame in which that segment lays in an isochronous subspace.

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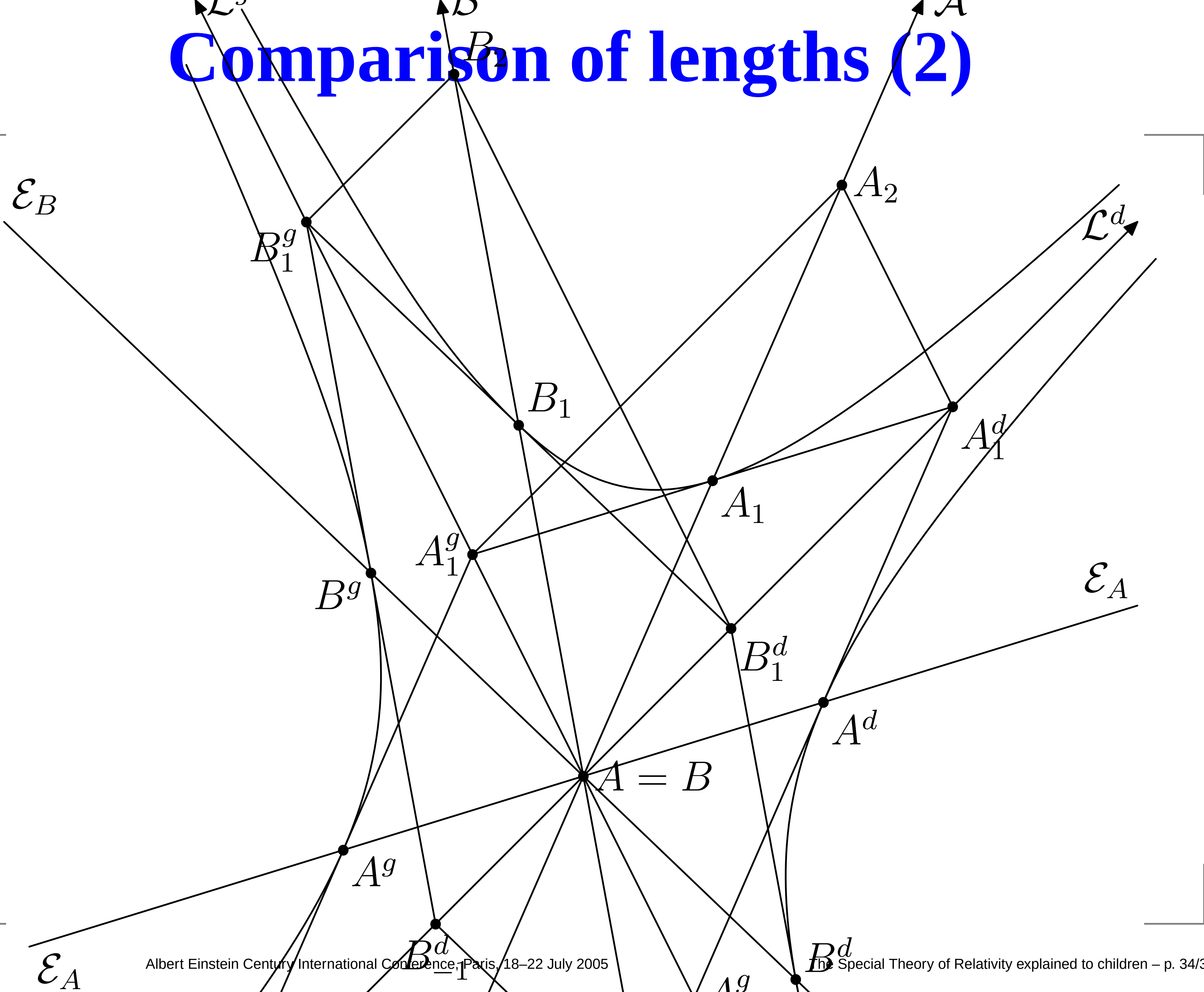
Two line segments $A\,A^d$ and $A\,B^d$ supported by two space-like straight lines $\mathcal{E}_A$ et $\mathcal{E}_B$, are of equal length if and only if A^d and B^d lie on the same hyperboloid with the light cone of A as asymptotic cone.

Comparison of lengths (2)



Conclusion

The comparison of time intervals and lengths presented above, founded on very simple geometric arguments, allows a very natural introduction of the pseudo-Euclidean metric of Minkowski's Space-Time. The construction of isochronous subspaces in two different Galilean frames leads to the formulas for Lorentz transformations with a minimum of calculations.

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The pictures above allow a very easy explanation of the apparent contraction of lengths and dilation of times associated to a change of Galilean frame.

Conclusion (2)

Given two events, with one in the future of the other, the straight line is the longest time-like path (when measured in proper time) which joins these two events. That property leads to a very simple explanation, without complicated calculations, of the (improperly called) paradox of Langevin's twins.

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By explaining that the affine structure of Space-Time should be questioned, by speaking about the gravity force and the equivalence principle, a smooth transition towards General Relativity, suitable from children 8 to 108 years old, seems possible.